VEHICLE RESPONSE AND PASSENGER COMFORT CONSIDERING NONLINEAR SUSPENSION AND RANDOM ROAD EXCITATION

By

B. SEETARAMA PATRO

DEPARTMENT OF MECHANICAL ENGINEERING

INDIAN INSTITUTE OF TECHNOLOGY KANPUR

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IN THE LOVING MEMORY

OF MY

RESPECTED FATHER

CERTIFICATE

This is to certify that the thesis entitled "Vehicle Response and Passenger Comfort Considering Nonlinear Suspension and Random Road Excitation" by B. Seetarama Patro is a record of work carried out under my supervision and has not been submitted elsewhere for a degree.

Birendra Sahay
B. SAHAY
Assistnat Professor
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur

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NOMENCLATURE

a_i	:	Slopes of nonlinear damping force vs velocity curve.
a, b	:	Numerical values of distance of axles from centre of gravity of sprung mass.
b_i	:	Intercept of damping force vs velocity curve.
B_1, B_2	:	Slopes of Power Spectrum on a log log plot.
C_i	:	Spring constant of ith tire (lb/ft.).
d_i	:	Damping coefficients of seats (lb./ft./sec.).
D_i	:	Equivalentlinear suspension damping coefficients (lb./ft./sec.).
D	:	Wheel base (ft.).
E	:	Expectation.
f	:	Frequency (cycle sec.)
f_a, f_b	:	Resultant forces on sprung mass due to road inputs and passengers.
f_U	:	Cut off frequency.
f_1, f_2	:	Tire force increments.
f_1', f_2'	:	Body force increments at support points.
g_i	:	Slopes of nonlinear damping force-velocity curve.
$G_X(f)$, $G_{XX}(f)$	:	One sided power spectral density (ft^2/cycle/ft.).
h	:	Sampling interval (ft.).
h_i	:	Absciassa of the points where damping force velocity curves intersect.
h (t)	:	Impulse response function.

$H(\omega)$	:	Complex frequency response.
i	:	Imaginary quantity.
J	:	1/2 pitch moment of inertia of sprung mass (slug ft^2).
K	:	Radius of gyration of sprung mass about its C.G. (ft.).
K_i	:	Spring constants of ith seat (lb/ft.).
l_1, l_2	:	Distances of seats from the C.G. of sprung mass (ft.).
L_1, L_2	:	Distance of axles from the C.G. of sprung mass (ft.).
m	:	Maximum lag number.
m_i	:	1/2 (mass of ith seat and passengers on it) (slug).
M	:	1/2 sprung mass (slug).
M_i	:	1/2 unsprung mass at ith axle (slug).
$p(\ )$	:	Probability density function.
q_a, q_b	:	Applied moments on sprung mass (lb. ft^2).
r	:	Lag number.
R	:	Inertia coupling ratio.
RS	:	Relative displacement between chassis and axle.
RW	:	Relative displacement between wheel and terrain.
R_i	:	Vertical deflection of ith wheel.
$R_x(\ )$, $R_{xx}(\ )$	:	Autocorrelation functions.
s	:	Laplace transform complex variable
s_i	:	Spring constants of suspension at ith wheel (lb./ft.)
$S_{xx}(\omega)$	:	Power spectrum (ft^2/rad./ft.)
$T(\ )$	:	Transfer function

v	:	Velocity of vehicle (ft./sec.).
X_i	:	Vertical position of terrain at ith wheel.
$Y(t)$	:	Bounce motion of sprung mass.
$Y_i(t)$	:	Vertical deflection at support points.
Z_i	:	Vertical deflection of ith seat.
λ	:	Wave length (ft.).
$\theta(t)$	:	Pitch deflect of sprung mass
Ω	:	Spatial frequency (rad./ft.)
$\omega, \omega_1, \omega_2,$	:	Circular frequency (rad./sec.)
ϕ_K	:	Power spectral density function (ft.^{2}/rad./ft.).
σ	:	Standard deviation.
ζ	:	Damping coefficient.
ρ	:	Ratio of two masses.
τ	:	Time difference.
$(\dot{\ })$	:	Differentiation with respect to time.
$(\bar{\ })$	:	Vector.
$[\ ]'$	:	Transpose of a matrix.
$[\ ]^*$	:	Complex conjugate transpose of a matrix.
$\lvert\ \rvert$	:	Modulus.

SYNOPSIS

Dissertation on

VEHICLE RESPONSE AND PASSENGER COMFORT CONSIDERING NONLINEAR SUSPENSION AND RANDOM ROAD EXCITATION

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by

B. SEETARAMA PATRO
Department of Mechanical Engineering
Indian Institute of Technology Kanpur
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The response of a vehicle as it moves on a rough road is investigated. For constant speed motion of the vehicle, the input is stationary. However some parameters of the vehicular system such as dashpot characteristics are nonlinear due to the inherent friction present. The equivalent linearization technique is applied to determine an equivalent linear damping which depend on the input to the system. This method is checked with the results obtained by numerical integration of system differential equations. For this purpose, random road data is generated from known power spectrum of the road profile.

The response quantities of interest are bounce and pitch accelerations, accelerations of seats and riders and displacements of sprung mass relative to unsprung masses. A method is suggested to select the parameters of the vehicular system such as to maximise the comfort of the rider.

CHAPTER I

INTRODUCTION

1.1 GENERAL

The importance of transport vehicles and its contribution to modern civilization can not be overemphasized. Therefore, the prime objective of the automotive engineer is to provide vehicles which are satisfactory and comfortable to the riders and to ensure safety of the cargo. The comfort of a rider depends on many factors such as noise, smell, mental condition of the rider, ability of the driver etc. and vibration to which he is subjected. The cargo safety can be precisely defined by putting a limit on the maximum displacement and/or acceleration of the cargo at specified frequencies.

So far no comprehensive criteria has been evolved to quantitatively determine the ride comfort taking into account the environmental conditions. Therefore, the designer has to resort to the evaluation of the dynamic response of the vehicle and the rider, when the vehicle moves on a road. The vehicle is subject to the following types of disturbance inputs :

a) Internal disturbances :

   i) Engine vibration

   ii) Vibration caused due to transmission parts such as gear box, propeller shaft etc.

   iii) Vibration caused by rotating parts such as wheels or eccentricity in bearings.

b) External disturbances :

i) Input from the road

ii) Aerodynamic forces and moments.

Engine vibration has been minimized by properly balancing the engine. Modern mounting techniques successfully reduce the engine vibration effects on the chassis. Precision manufacture reduces eccentricity in bearings. There is inherent imbalance in tyres, as portion of the wheel in contact with the ground, support the weight of the vehicle and thus gets flattened. With a tyre under suitable inflation pressure, this is not much of a problem. Under normal environmental conditions and low speeds normally encountered on Indian roads, aerodynamic forces can mostly be taken care of by the proper design of the profile of the body of the car.

The major cause of vehicle vibration is due to the road roughness. By road roughness we do not mean the texture roughness of the road, but the waviness of the road. It is not possible to lay a perfect straight road. As the vehicle moves on these roads its wheels sense the waviness. The disturbing input to the vehicle depends on type of road roughness as well as on vehicle speed.

The excitation to the vehicle is random in nature. This excitation is filtered through tyres and suspension mechanism. The suspension characteristics should be such as to produce a response which should not be excessive, causing fatigue and uneasiness to the passenger over a specified period of time.

1.2 LITERATURE SURVEY

Random nature of road roughness is assumed to be stationary, ergodic [1,2,3,4] having a Gaussian distribution. Thus it is only reasonable to describe the elevation of road surface statistically such as in terms of power spectral density. If the profiles of two long stretches of a road are taken and analyzed, the power spectral density curves in both the cases should be approximately the same to support the assumption of the stationarity of the profile. In practice, the power spectral density curves match at long wavelengths, but a large difference between the two curves is observed at short wave lengths. Jones has been critical of this phenomenon called "tail inconsistencies" [1]. Srikantaiah [2] has measured runway roughness by two methods :

a) Surveying

b) Profilometer, designed by Panchal.

He has observed that roughness data obtained from surveying is good for long wavelengths where as data from profilometer measurement is good for short wavelengths.

Power spectrum of road roughness when plotted on a log log scale looks like a straight line [3,4]. Macaulay has suggested to use two straight lines to describe the power spectral density. At higher wavelengths the slope is higher [3]. The discontinuity between two straight lines occurs at a wave length of approximately 20 feet. This is one of the standard data used in road construction.

Dodds and Robson have classified roads according to their power spectral characteristics and have noted the corresponding equations for power spectrum [4].

In early fifties response of vehicle moving on a rough road has been investigated using determinastic approach [1]. The usefulness of these investigations are limited because the actual terrain has a random profile. The work were mainly confined to the analysis of simplified models operating on geometrically idealised surfaces whereas the recent trend is in analysing more complex multi-degree of freedom systems operating on an arbitrary terrain profile. This transition has been made possible mainly due to computers.

The vehicle is represented by a set of differential equations. These equations can be developed either by Lagrange's or Newton's approach. The system can be simulated on analog computer with known dynamic equations and given road conditions [5]. However for nonlinear systems the proper tool is a digital computer where the differential equations can be numerically integrated [1]. This method is commonly known as time domain approach, where the bounce and pitch motions of the vehicle and rider are described as a function of time. It gives satisfactory results but great deal of time is spent in programming and running the simulation either on analog or digital computers [6].

An alternate approach is to transfer the system differential equation representing the vehicle to frequency response

function [6]. The forcing function is then described statistically in the form of power spectral density. This method is known as frequency domain analysis where frequency is the independent variable. The differential equations are converted to algebraic equations resulting in decrease of computational time. This has the added advantage that it takes into account the randomness of road profile.

The input to the vehicle depends on the road roughness and on the vehicle speed. When the vehicle moves at a constant speed on a road having stationary property, the input to the vehicle is stationary. However, if the vehicle moves at a variable speed, the vehicle is subject to a nonstationary input even though the road roughness may be stationary. Virchis [7] has shown that the effect of nonstationarity due to commonly encountered accelerations and decelerations which are not more than 1/2 g for road vehicles, can be neglected in predicting the vehicle response. The simplification is very much valuable because the analysis is presumably applicable not only to uniform accelerations and decelerations but also to nonuniform accelerations and decelerations.

The work done so far in the field of vehicle vibration assumes the system parameters to be linear. Although tyre and suspension spring stiffness have linear characteristic in the working range, the damping consists of linear viscous damping and Coulomb damping due to friction in the dashpot. Ellis suggests to include the effect of Coulomb damping for future work [5]. In this

connection, the data supplied by M/s Hindustan Motors Ltd., Calcutta, shows the nonlinearity of dashpot characteristics [8].

For nonlinear systems with harmonic input, one can use describing function approach, the theoretical basis of which lies in the method of slowly varying parameters and equivalent linearization suggested by Krylov and Bogoliubov [9,10]. It has also been suggested to find out equivalent linear damping taking into account of the energy absorbed in one cycle due to friction in a dashpot [11]. For random inputs these methods can not be directly applied.

Ariaratanam has used the theory of Markov random process and the associated Fokker-Planck equation to solve nonlinear two degree of freedom random vibration problems [12]. Crandall has obtained exact solution of a single degree freedom system by means of the Fokker-Planck equation [13]. The result is compared with approximate solution obtained from the application of equivalent linearization technique. They agree up to first order of nonlinearity. Crandell has applied perturbation method to problems in which the nonlinear terms in the governing equations are small [14]. Duffing's type nonlinearity can be better handled in this technique. These methods are apparently difficult to apply to multi-degree of freedom systems. Even for two degree of freedom problems, in most cases, it is not possible to integrate the Fokker-Planck equation to get an exact solution.

The method of describing function has been extended to systems with random input [10]. The treatment is similar to the equivalent linearization technique developed by Caughy [15]. These methods belong to the class where the nature of the solution is assumed a priori. These methods are also applicable to systems with nonlinear parameters which are not symmetric about the origin. Equivalent linearization technique like the application of Fokker-Planck equation or perturbation method is difficult to apply to multi-degree of freedom systems.

The effect of various parameters on random vibration isolation problem can be better handled if the number of degrees of freedom can be reduced. This can be achieved by "elastic decoupling" and "inertia decoupling" of the system [16]. The system with four degrees of freedom and two inputs is split into two subsystems each having two degrees of freedom with a single input. The response of such subsystems when subject to white noise input and ramp noise input have been extensively studied [17,18].

For multi-input, multi-output systems, the matrix of cross spectral densities of the response can be evaluated from the frequency response function and matrix of cross spectral density of the excitation [14]. The response of the vehicular system should be such as to provide a satisfactory ride.

Early indices proposed for evaluating ride comfort were incomplete in their treatment of human response to cumulative effect

of frequencies, amplitudes of accelerations, velocity, displacement and phase. The establishment of uniform scale for ride comfort is complicated because of physical nonuniformity of test subject's reactions to vibration. Janeway divided the frequency band into three sections from 1 to 6, 6 to 20 and 20 to 60 cycles per second [19]. He stated that for the lowest band the rate of change of acceleration or jerk value was the criterion where as for the middle band the acceleration and for the upper band the velocity should be the comfort criterion. In 1958 Dieckmann proposed an empirical method of assessing the degree of ride comfort and broke the frequency band into three sections. He suggested acceleration, velocity and displacement were criteria for three bands respectively [19]. Mitschke has concluded that the acceleration should be small for superior ride comfort and ride safety [19]. There are three simple criteria after Goldman for subjective response to shock and vibration; the threshold of perception, of unpleasantness and of tolerance [20]. The latter two are generally difficult to identify and reproduce.

The tests for ride comfort consists in finding the response of a human being to a harmonic input. But in practice the human being is subject to vibrations of varying amplitudes at varying frequencies and phase. To solve such problems some guide lines are recommended to determine square root of the summation of the squares of ride comfort indices taken for different component vibrations [1].

The approach using the index of ride comfort assumes that the contributions of vibration to ride comfort at varying frequencies are same. This is far from truth. This is because the mechanical impedence of man varies for different frequencies. At low frequencies (1 to 100 cps) the body acts as a combination of spring mass damper system [20]. For a sitting man the first resonance is between 4 to 6 cps. Between 20 to 30 cps the head exhibits mechanical resonance and frequency range of 60 to 90 cps suggests eyeball resonance. Impedence depends also, on whether the rider is sitting erect or relaxed. Similarly the resonant frequencies are different for a standing man or a rider in supine position. Therefore, it is only natural to have weighting function as a function of frequency to calculate the contribution of vibration to ride comfort.

Butkunas translated the input spectrum into spectrum of sensation through human body defining the transfer function as the ratio of ride sensation and input spectrum [1]. Butkunas's work is based on Goldman's criteria of comfort. This scheme has obvious limitations because neither there is a single input to the rider nor the human system is linear. Vibration affects the rider through seperate inputs such as hand on steering wheel, feet on chassis floor and back and head on the backrest and headrest respectively.

Ride comfort depends on the total time a person stays in the moving vehicle [21]. For a shorter duration one can tolerate

vibrations of greater amplitudes. Based on this fact, the International Standard Organisation (I.S.O.) approved a recommendation concerning vibration limits and human being. For vertical vibrations the recommendation is shown in the Figure (12) which indicates a maximum sensitivity to vibration in the frequency range of major body resonances. The amplitude of vertical acceleration is clearly frequency and time dependent.

1.3 PRESENT INVESTIGATION

It has been emphasized in earlier publications [1,5] that the effect of friction in dashpots, used in the suspension mechanism of a vehicle should be taken into account. From the experimental data supplied by M/s Hindustan Motors Ltd. [8], it is evident that friction gives rise to nonlinear characteristics of the dashpot response. Thus the differential equations governing the dynamics of the vehicular system becomes nonlinear and coupled. Equivalent linearization technique using the minimum mean squared error criteria is applied to determine the equivalent linear parameters of the system so that the system will give approximately same response as would be given by a nonlinear system.

The input to the system is described by a power spectrum. As data for Indian roads were not available, the data of a runway is used to calculate the power spectrum. Dodds and Robson [4] have described different types of roads based on their roughness and their corresponding power spectrum. These are used as the input to the system.

A model for the vehicle is developed. The model has several parameters. The system is decoupled statically and dynamically to two subsystems. Frequency response functions are determined for 2 and 6 degrees of freedom systems. With known complex frequency response and input power spectrum, the power spectrum of different response quantities is determined. The response of the seats in a vehicle act as input to the rider.

Human being act as a linear system under low frequency vibration conditions [20] . The discomfort of human being is frequency dependent. The resonant frequencies depend on posture of the rider such as sitting, standing etc. A transfer function of human system has been derived from the I.S.O. recommendation for effects of vibration on man. Using this frequency dependent transfer function the response of the rider to bounce and pitch motions of the vehicle are determined. The response gives an idea of whether the rider is on threshold of perception or in the intolerance range. Comfort is a function of the r.m.s. acceleration of the subject.

There are several system parameters which can be varied to obtain an optimum condition resulting in best possible comfort to the rider in a vehicle moving over a given rough road. The criteria for selecting the system parameters is to have minimum r.m.s. acceleration of vibration of the rider subject to a limited wheel excursion.

The results obtained by using equivalent linearization technique is verified by numerical integration of system differential

equations. For this purpose random data is generated from a known power spectrum. These data are used as inputs to the system. Gill's modification of Runge Kutta method is used for numerical integration purpose.

The thesis ends with the discussion of the problem. Several conclusions are drawn from this investigation. Some suggestions are made for future work.

CHAPTER II

ROAD ROUGHNESS

2.1 SPECTRAL DECOMPOSITION OF ROAD ROUGHNESS

2.1.1 Introduction

The autocorrelation function $R_{xx}(t_1, t_2)$ of a random process $X(t)$ is defined as

$$R_{xx}(t_1, t_2) = E\left[X(t_1)\, X(t_2)\right] \tag{2.1}$$

For a stationary random process the autocorrelation function is a function of difference of time t_1 and t_2. By definition, this function is symmetric.

$$R_{xx}(\tau) = R_{xx}(-\tau) \tag{2.2}$$

Where $\tau = t_2 - t_1$. For an arbitrary function $h(t)$ the autocorrelation function satisfies [14].

$$R_{xx}(t_j, t_k)\, h(t_j)\, h^*(t_k) \geqslant 0 \tag{2.3}$$

A function such as $R_{xx}(t_1, t_2)$ which satisfies equation (2.3) is said to be nonnegative definite. A theorem due to S. Bochner [14] asserts that every nonnegative definite function has a nonnegative Fourier transform, if such a transform exists, that is,

$$S_{xx}(\omega) = \frac{1}{2\pi} \int_{-\infty}^{\infty} R_{xx}(\tau) \exp(-i\omega\tau)\, d\tau \geqslant 0 \tag{2.4}$$

From the Fourier inversion formula it follows that $R_{xx}(\tau)$ can be expressed in terms of $S_{xx}(\omega)$:

$$R_{xx}(\tau) = \int_{-\infty}^{\infty} S_{xx}(\omega) \exp(i\omega\tau)\, d\omega \tag{2.5}$$

Equations (2.4) and (2.5) are popularly known as Weiner-Khintchine relations. A physical meaning can be given to $S_{xx}(\omega)$ by considering the limiting case of equation (2.4) in which $\tau = 0$.

$$R_{xx}(0) = E\left[X^2\right] = \int_{-\infty}^{\infty} S(\omega)\, d\omega \tag{2.6}$$

The mean square of the process equals to the sum over all frequencies of $S_{xx}(\omega)$ so that $S_{xx}(\omega)$ can be interpreted as a mean square spectral density. Since the electrical engineers were the first to usefully exploit this, they interpreted $S_{xx}(\omega)$ as power contained in a wave and gave an alternate name as power spectrum. This concept can be analogously carried over to mechanical systems. For instance, if $X(t)$ is the displacement, then $E\left[X^2(t)\right]$ is proportional to the average strain energy and if $X(t)$ represents the velocity, then $E\left[X^2(t)\right]$ is proportional to the average kinetic energy and $S_{xx}(\omega)$ describe the distribution of the total mean square value over the frequency domain.

Spectral density $S_{xx}(\omega)$ is an even function. In practice frequency is only positive and expressed in cycles per second rather than radians per second. The experimental spectral density is denoted by $G_{xx}(f)$. The relation between $G_{xx}(f)$ and $S_{xx}(\omega)$ is given by,

$$G_{xx}(f) = 4\pi S_{xx}(\omega) \tag{2.7}$$

In place of (2.6) we will have,

$$R_{xx}(0) = E\left[X^2\right] = \int_{0}^{\infty} G_{xx}(f)\, df \tag{2.8}$$

2.1.2 Measurement of Road Roughness

Prior to deciding the method to be used for measuring the road roughness, it is necessary to know the frequency range of interest for power spectrum calculations. For the human vibration problem the frequency range of interest is 1 cps to 80 cps. If the vehicle vibration problem is investigated for the vehicle moving at a speed of 30 to 75 mph., the wavelength of interest can be calculated from the following relation :

$$f = v / \lambda \tag{2.9}$$

where v = vehicle speed (ft./sec.)

and λ = wave length (ft.)

For the frequency range and vehicle speed mentioned, the minimum wave length of interest is about 1/2 ft. and maximum 110 ft.

One of the most commonly used methods of measuring road surface is surveying by level and staffs. If sampling, which is defined as the points at which data are observed, is too close together, it will yield correlated and highly redundant data. This results in wastage of time in surveying as well as computation. If sampling is too far apart, the points will contain the amplitudes not only of the wave lengths, we think we are analysing, but of other harmonics as well. This phenomenon is known as "alising". If the roughness in the higher harmonics is relatively small, this need not matter greatly, but if the roughness is relatively large it can cause

misleading results. A sampling theorem credited to Shannon states that the highest frequency that may be observed by sampling a wave at a discrete interval Δ is equal to $1/2\Delta$, known as Nyquist frequency [1, 22]. Therefore, the interval that should be employed for sampling is equal to 1/4 ft in order to avoid alising.

The sampling interval of 1/4 ft. is too close to be obtained in surveying. Therefore, for most of the roads and airfields runway roughness survey up to date, the interval is 2 ft. [22]. Obviously this method gives an incomplete picture of profile roughness and has to be suplemented by some other method. Many methods of measuring road roughness have been listed by Macaulay [3]. Srikantaiah [2] has used Panchal's profilometer as a suplement.

2.1.3 Calculation of Power Spectral Density Function

The random road profile roughness, besides being stationary, is assumed to be weakly ergodic. This implies that temporal averages can replace the ensemble averages. This is very useful because from a single long record of roughness data, the mean and autocorrelation function can be calculated. This avoids the need for collection of a huge set of data. For calculation of power spectral density function, the autocorrelation function need to be computed first.

For N data values $\{Y'_n\}$, n = 1, 2, ..., N, the autocorrelation function, R_r, at the displacement rh is calculated [23]:

$$R_r = R_y(rh) = \frac{1}{N-r} \sum_{n=1}^{N-r} Y'_n \; Y'_{n+r}, \; r = 0, 1, 2, \ldots, m \tag{2.10}$$

Where r is lag number, m is maximum lag number, h is the sampling interval and Y'_n is the road height at a point n , measured over a certain mean. Y'_n is calculated from the following relation.

$$Y'_n = Y_n - \frac{1}{N} \sum_{n=1}^{N} Y_n, \; n = 1, 2, \ldots, N \tag{2.11}$$

Where Y_n is the road height read off over a datum at a point n . The second term on right hand side of equation (2.11) denotes the mean of $\{Y_n\}$, n = 1, 2, ... , N.

A procedure recommended in spectral analysis technique is prewhitening [23]. The process of prewhitening amounts to applying a special digital filter to the data which will result in the filtered data having a flat spectrum. The digital filter used is given by,

$$Y'_n(X) = Y_n(X) - Y_n(X - h) \tag{2.12}$$

The initial or "raw" estimate of the power spectral density function are calculated from the following relation :

$$\phi_k = \phi\left(\frac{\pi k}{mh}\right) = \frac{h}{\pi}\left[R_o + 2 \sum_{r=1}^{m-1} R_r \cos\left(\frac{\pi r k}{m}\right) + (-1)^k R_m\right]$$

$$\text{for } k = 0, 1, 2, \ldots, m \tag{2.13}$$

Where $\phi_k = \phi(\Omega)$; $\Omega = \frac{\pi k}{mh}$, k = 0, 1, 2, ..., m and $\Omega = \frac{\pi}{h}$ corresponds to cut off frequency.

The raw estimate of the power spectrum are then smoothened by "hamming" [24].

$$\phi'_o = .54\ \phi_o + .46\ \phi_1$$

$$\phi'_k = .23\ \phi_{k-1} + .54\ \phi_k + .23\ \phi_{k+1}$$

$$\phi'_m = .54\ \phi_m + .46\ \phi_{m-1} \qquad (2.14)$$

A check sum is computed to check the computations of the estimates [24]:

$$\text{CHKSUM} = \frac{\pi}{mh} \left[\frac{1}{2} \left(\phi'_o + \phi'_m \right) + \sum_{k=1}^{m-1} \phi'_k \right] \qquad (2.15)$$

To compensate for prewhitening, the smoothened spectrum is recoloured by the following operation [24]:

$$\phi_k = \phi'_k / 2 \quad (1 - \cos \Omega h) \qquad (2.16)$$

2.1.4 Results

The power spectral density is not plotted against wave length but spatial frequency Ω (radians/ft). This is useful because to obtain the input to the vehicle in radians per second, it is only necessary to multiply the road frequency by the vehicle speed v (ft./sec.). Similarly the amplitude of power spectral density function is expressed in ft^2/rad/ft., since to obtain the average amplitude in any frequency band of interest, it is only necessary to measure the area under the curve between the upper and lower frequency limits.

The data for Indian roads are not available. To calculate the power spectral density function, data is taken for a taxi way [22]. The data is given for 1501 points at an interval of 2 ft. The power spectral density is calculated and plotted in Figure (1). The power spectrum is also calculated for the two 1500 ft. intervals and also plotted in Figure (1).

2.1.5 Discussion

It is seen from Figure (1) that the power spectrums of the two stretches of road profiles are quite close at low frequency range. The difference becomes large at higher frequencies. This is described by Jones as "tail inconsistency" [1]. The fact that power spectrums are almost same in the large range of frequencies confirms stationarity of the road profile. From the plot of power spectrum on a log-log scale (Figure 2), it is observed that the power spectrum is approximately a straight line.

Since the measurements available are at 2 ft. interval, consistent result is expected upto a wavelength of 4 ft. (using Shannon's Theorem). But for the study of vehicle vibration, we need the measurements of the road profile at 1/4 ft. intervals as calculated earlier in article 2.1.2. Thus the power spectrum plotted in Figure (2) is incomplete, so far as study of vehicle vibration is concerned and the data cannot be used for the investigation. If a large set of road profile data is available using a calibrated

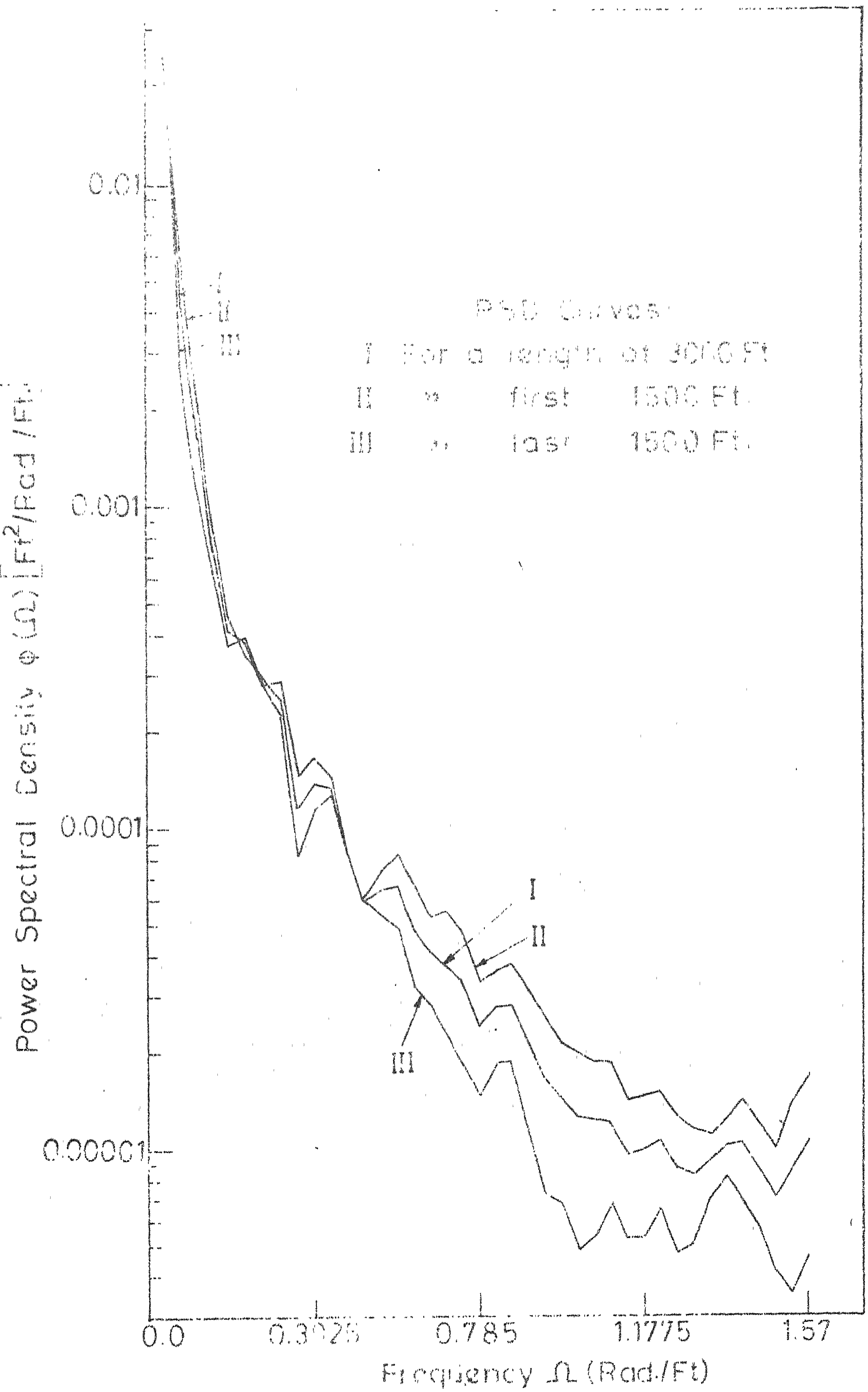

FIG. 1 POWER SPECTRAL DENSITY OF RUNWAY ROUGHNESS

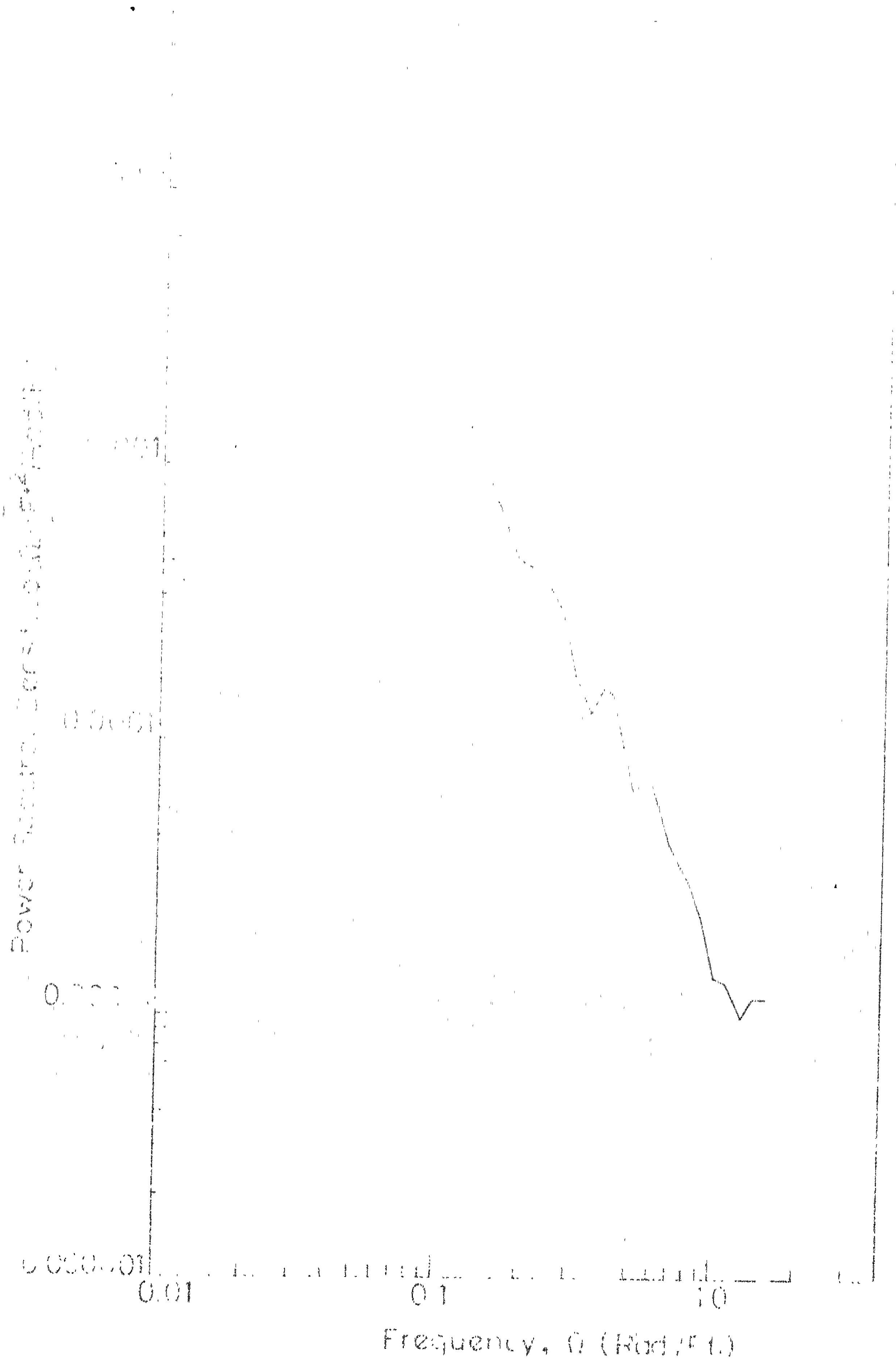

FIG 2 POWER SPECTRAL DENSITY OF RUNWAY ROUGHNESS.

profilometer, it can be effectively used for calculation of vehicle response.

Profile data for Indian roads are not available. Therefore the power spectrum data for this investigation is taken from the one given by Dodds and Robson [4]. They have surveyed different European roads and have classified them according to roughness. The power spectrum consists of two straight lines meeting at a wavelength of 6.3 metres and are defined by the formula,

$$G(n) = \begin{cases} G(n_o) \ (n / n_o)^{-B_1} & n \leqslant n_o \\ G(n_o) \ (n / n_o)^{-B_2} & n \geqslant n_o \end{cases} \tag{2.17}$$

where $n_o = 1 / 6.3$ (cycles/m.)

$G(n_o)$ is the roughness coefficient (m^2/cycle/m)

B_1, B_2 are slopes of power spectrum on a log log plot.

The power spectrum data for different roads are given below

Road Class		$G(n_o) / 10^{-6}$	B_1		B_2	
			Mean	S.D.	Mean	S.D
Motor ways	Very Good	2–8	1.945	.464	1.360	.221
	Good	8–32				
Principal Roads	Very Good	2–8	2.05	.487	1.44	.266
	Good	8–32				
	Average	32–128				
	Poor	128–512				
Minor Roads	Average	32–128	2.28	.534	1.428	.263
	Poor	128–512				
	Very Poor	512–2048				

2.2 CORRELATION OF THE INPUTS

The vehicle has inputs depending on the number of wheels. By the assumption that both front and rear wheels are subject to the same road roughness, the number of inputs becomes two for a vehicle with four wheels. If it is further assumed that rear wheel follows the front wheel, then the input spectrum to both the wheels are the same and at a particular instant inputs to the two wheels are related. The relation depends on the wheel base and vehicle speed. Clearly the input to the rear wheel lags the input to the front wheel by a time τ_{ik} given by [6],

$$\begin{aligned} \tau_{ik} &= \frac{d}{v}, \; i \neq k \\ &= 0, \; i = k \end{aligned} \tag{2.18}$$

Where d is wheel base (ft.)

The cross-correlation function for the two inputs are,

$$R_{X_iX_k}(\tau) = R_{XX}(\tau - \tau_{ik}) \tag{2.19}$$

By the application of (2.4) we get the cross power spectral density,

$$\begin{aligned} S_{X_iX_k}(\omega) &= \int_{-\infty}^{\infty} R_{X_iX_k}(\tau) \exp(-i\omega\tau)\, d\tau \\ &= \int_{-\infty}^{\infty} R_{XX}(\tau - \tau_{ik}) \exp(-i\omega\tau)\, d\tau \end{aligned}$$

Letting $t = \tau - \tau_{ik}$

$$S_{X_iX_k}(\omega) = \int_{-\infty}^{\infty} R_{XX}(t) \exp(-i\omega t) \exp(-i\,\omega\tau_{ik})\, dt \tag{2.20}$$

Substituting equation (2.5) in (2.20)

$$S_{X_iX_k}(\omega) = S_{XX}(\omega)\exp(-i\omega\tau_{ik}) \qquad (2.21)$$

2.3 RELATIONSHIP BETWEEN DISPLACEMENT, VELOCITY AND ACCELERATION POWER SPECTRUMS OF THE INPUT

For a stationary random process,

$$R_{XX}(\tau) = R_{XX}(t-s) = E\left[X(t)\,X(s)\right] \qquad (2.22)$$

Differentiating the equation (2.22) with respect to t and s and noting that differentiation and expectation commute,

$$\frac{\partial^2}{\partial t\,\partial s} R_{XX}(\tau) = \frac{\partial^2}{\partial t\,\partial s} E\left[X(t)\,X(s)\right]$$
$$= E\left[\dot{X}(t)\,\dot{X}(s)\right] = R_{\dot{X}\dot{X}}(\tau) \qquad (2.23)$$

Differentiating equation (2.5) with respect to t and s

$$\frac{\partial^2}{\partial t\,\partial s} R_{XX}(\tau) = \frac{\partial^2}{\partial t\,\partial s}\int_{-\infty}^{\infty} S_{XX}(\omega)\exp\left\{i\omega(t-s)\right\}d\omega$$
$$= -i^2\omega^2\int_{-\infty}^{\infty} S_{XX}(\omega)\exp\left\{i\omega(t-s)\right\}d\omega$$
$$= \omega^2 R_{XX}(\tau) \qquad (2.24)$$

As in equations (2.4) and (2.5) the Weiner-Khintchine relation for $\dot{X}$ can be written as

$$S_{\dot{X}\dot{X}}(\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} R_{\dot{X}\dot{X}}(\tau)\exp(-i\omega\tau)\,d\tau$$
$$R_{\dot{X}\dot{X}}(\tau) = \int_{-\infty}^{\infty} S_{\dot{X}\dot{X}}(\omega)\exp(i\omega\tau)\,d\omega \qquad (2.25)$$

By taking inverse Fourier transform of equation (2.23) and using relations (2.24) and (2.25),

$$\int_{-\infty}^{\infty} S_{\ddot{X}\ddot{X}}(\omega) \exp(i\omega\tau) = \int_{-\infty}^{\infty} \omega^2 S_{XX}(\omega) \exp(i\omega\tau)\, d\omega$$

Or

$$S_{\ddot{X}\ddot{X}}(\omega) = \omega^2 S_{XX}(\omega) \qquad (2.26)$$

Similarly, it can be shown that,

$$S_{\dddot{X}\dddot{X}}(\omega) = \omega^4 S_{XX}(\omega) \qquad (2.27)$$

2.4 GENERATION OF RANDOM DATA

To generate random road data, at some specified intervals, which corresponds to a given power spectrum, we use a method as described below [25],

$$\text{Let } X(t) = \sum_{k=1}^{N} A_k \operatorname{Cos}(2\pi f_k t + 2\pi \phi_k) \qquad (2.28)$$

Where ϕ_k is the Kth sample value of uniformly distributed random variable in the interval $(0, 1]$ that is,

$$f_\phi(x) = 1 \quad \text{if} \quad x \in (0, 1]$$

$$= 0, \text{ otherwise} \qquad (2.29)$$

$$f_k = (k - \tfrac{1}{2})\, \Delta f \qquad (2.30)$$

and A_k are constants depending on the power spectrum.

$$E\left[X(t)\right] = \sum_{k=1}^{N} A_k\, E\left[\operatorname{Cos}\left\{2\pi (f_k t + \phi_k)\right\}\right]$$

Now,

$$E\left[\operatorname{Cos}\left\{2\pi (f_k t + \phi_k)\right\}\right]$$

$$= \int_0^1 \operatorname{Cos}\left\{2\pi (f_k t + \phi)\right\} d\phi = 0$$

Therefore, $E\left[X(t)\right] = 0$ (2.31)

$$E\left[X(t)\ X(t+\tau)\right] = \sum_{j=1}^{N}\sum_{k=1}^{N} A_j A_k E\left[\operatorname{Cos}\left\{2\pi f_k (t+\tau) + 2\pi \phi_k\right\} \operatorname{Cos}\left\{2\pi f_j t + 2\pi \phi_j\right\}\right]$$

Now let,

$$I = E\left[\operatorname{Cos}\left\{2\pi f_k (t+\tau) + 2\pi \phi_k\right\} \operatorname{Cos}\left\{2\pi f_j t + 2\pi \phi_j\right\}\right]$$

Case I :

If $j = k$,

$$I = \int_0^1 \operatorname{Cos}\left\{2\pi f_j (t+\tau) + 2\pi \phi_j\right\} \operatorname{Cos}\left\{2\pi f_j t + 2\pi \phi_j\right\} d\phi$$

$$= \frac{\operatorname{Cos}(2\pi \tau f_j)}{2} \qquad (2.32)$$

Case II :

If $j \neq k$, $I = 0$

Therefore, $E\left[X(t)\ X(t+\tau)\right] = R_X(\tau)$

$$= \sum_{j=1}^{N} \frac{A_j^2 \operatorname{Cos}(2\pi f_j \tau)}{2} \qquad (2.33)$$

Weiner - Khintchine relationship can be written in the form,

$$G_X(f) = 4\int_0^{\infty} R_X(\tau) \operatorname{Cos}(2\pi f \tau)\, d\tau \quad \text{for } 0 < f < \infty \qquad (2.34)$$

$$R_X(\tau) = \int_0^{\infty} G_X(f) \operatorname{Cos}(2\pi f \tau)\, df \qquad (2.35)$$

From equations (2.33) and (2.35),

$$\sum_{j=1}^{N} \frac{A_j^2}{2} \operatorname{Cos}(2\pi f_j \tau) = \int_0^{\infty} G_X(f) \operatorname{Cos}(2\pi f \tau)\, df \qquad (2.36)$$

$$\text{Or} \quad \frac{A_j^{\,2}}{2} = G_X(f_j)\ \Delta f$$

$$\text{Or} \quad A_j = \left[2\, G_X(f_j)\ \Delta f\right]^{1/2} \tag{2.37}$$

Δf must be small, such that A_j will represent the power spectrum correctly. Let f_U be the cut off frequency beyond which the one sided power spectrum is insignificant. N can be found out from the following relation :

$$N\ \Delta f = f_U \tag{2.38}$$

Using equations (2.28), (2.29), (2.30) and (2.37) the random road data can be generated for a known power spectrum.

CHAPTER III

MATHEMATICAL MODEL OF THE VEHICLE

3.1 INTRODUCTION

An idealized mathematical model of the vehicle is necessary in order to study the dynamic response of the vehicle subject to random road input. Representative model has obvious advantages over the alternative approach of manufacturing the vehicle and then conducting physical tests. A model which is too simple can hardly predict the actual response of the system. On the other hand a highly complicated model becomes very difficult to analyse. The automotive engineer has to draw a compromise and select a model representing the vehicle for the determination of responses.

3.2 ASSUMPTIONS

Certain assumptions are necessary to make a mathematical model from the physical system. The assumptions make the mathematical model easier to handle and approximately represent the physical system in the range of inputs that are usually expected in the system. These assumptions are :

a) The road is hard and does not deform when vehicle passes over it.

b) There is no loss in tyre contact with the road. If the vehicle loses contact with the ground, it falls to the ground due to the gravitational force. For roads, encountered, loss of tyre contact with the road is not a

frequent phenomenon. Under this assumption the vehicle tyre follows the road profile closely and gravitational force does not play part in the governing differential equations.

c) There is bounce and pitch motions of the vehicle due to different inputs at front and rear wheels. Even for the front two wheels the input may be different. But the difference is small compared to the expected difference of inputs between the front and rear wheels. Another contributing factor which suggests to neglect the roll motion is the fact, that the suspension springs and dashpots are not located directly above the wheels, but towards the centre of the vehicle.

d) The chassis is assumed to be rigid.

e) The damping in tyre is neglected.

f) The tyre and suspension stiffness characterstics are measured by static tests. The stiffness is assumed to remain the same under the dynamic load conditions. Ellis [5] has observed that the dynamic spring constants are less than the static spring constants, which implies that the natural frequencies at dynamic condition is less than that at the static condition.

g) The pitch axis is assumed to remain stationary with respect to the chassis. The vehicle has the pitch motion about the centre of gravity of the spring mass [26].

3.3 BASIC EQUATIONS

The inputs to the model as shown in figure (3), are vertical road displacements X_1 and X_2. The bounce and pitch motions are affected by the load (number of passengers, baggage etc.) on the vehicle. Horizontal disturbing forces due to wind gust etc. have negligible effect on bounce and pitch motions. The relationship between vertical displacements Y_1, Y_2 of the support points and bounce and pitch motions are given by,

$$Y_1 = Y + L_1\,\theta \tag{3.1}$$

$$Y_2 = Y + L_2\,\theta \tag{3.2}$$

$$\text{where } L_1 = a$$

$$\text{and } L_2 = -b \tag{3.3}$$

The resultant forces and moments due to forces on the supports,

$$f_a = f_1' + f_2'$$

$$q_a = L_1\,f_1' + L_2\,f_2' \tag{3.5}$$

Resolving the forces acting on the sprung mass and taking moments about the centre of gravity,

$$M\,\ddot{Y} = f_a - f_b \tag{3.6}$$

$$J\,\ddot{\theta} = q_a - q_b \tag{3.7}$$

For the front suspension,

$$M_1\,\ddot{R}_1 = f_1 - f_1' \tag{3.8}$$

where,

$$f_1 = C_1\,(X_1 - R_1) \tag{3.9}$$

and

$$f_1' = S_1\,(R_1 - Y_1) + D_1\,(\dot{R}_1 - \dot{Y}_1) \tag{3.10}$$

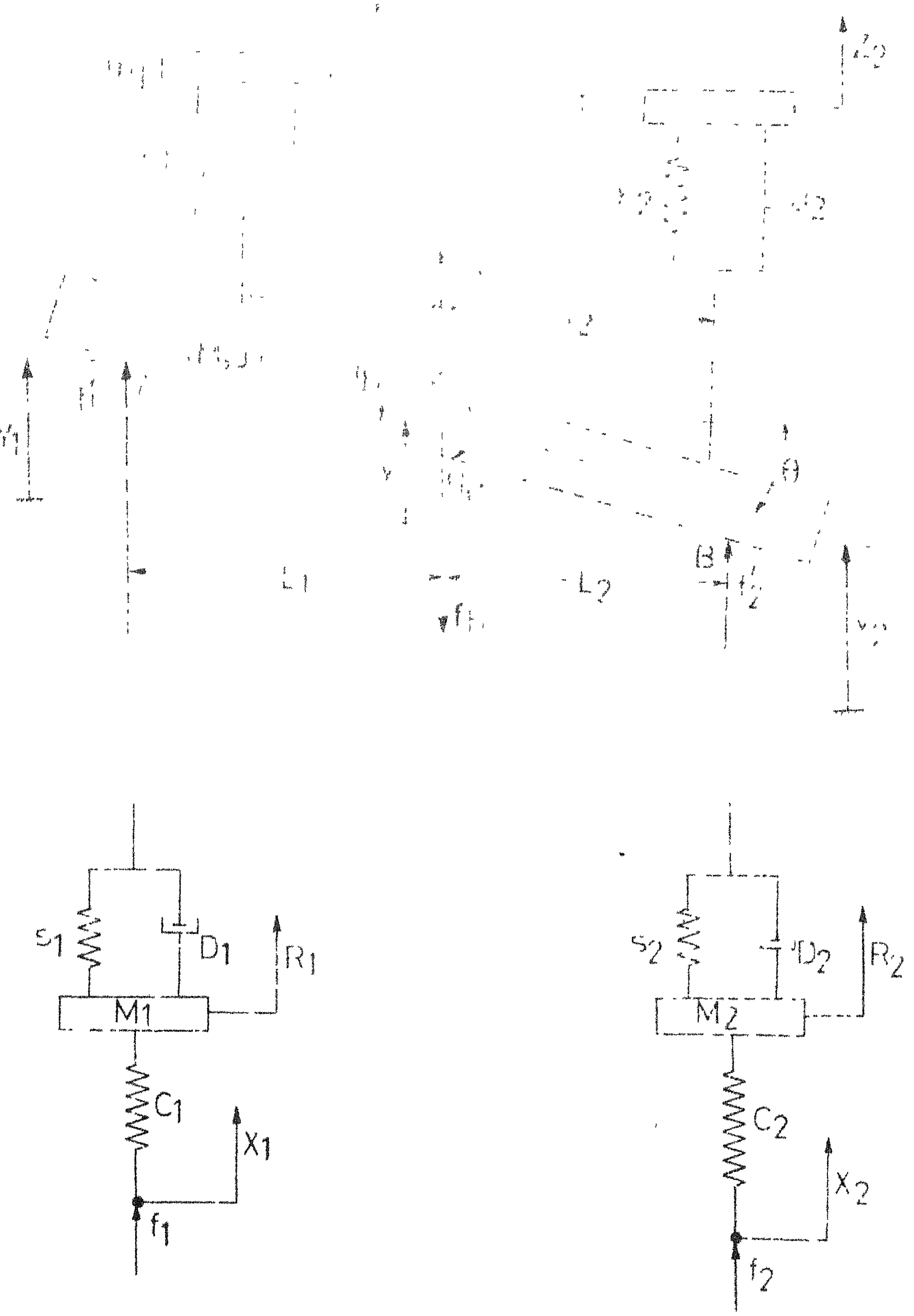

FIG 3 MODEL OF THE VEHICLE

Similarly, for the rear suspension,

$$M_2 \ddot{R}_2 = f_2 - f_2' \tag{3.11}$$

where,

$$f_2 = C_2 (X_2 - R_2) \tag{3.12}$$

and $$f_2' = s_2 (R_2 - Y_2) + D_2 (\dot{R}_2 - \dot{Y}_2) \tag{3.13}$$

The equations of motion for the seats and passengers are given by,

$$m_1 \ddot{Z}_1 (t) = -K_1 (Z_1 - Y - l_1 \theta) - d_1 (\dot{Z}_1 - \dot{Y} - l_1 \dot{\theta}) \tag{3.14}$$

and $$m_2 \ddot{Z}_2 (t) = -K_2 (Z_2 - Y - l_2 \theta) - d_2 (\dot{Z}_2 - \dot{Y} - l_2 \dot{\theta}) \tag{3.15}$$

The force and moments due to loads are given by,

$$f_b = - K_1 (Z_1 - Y - l_1 \theta) - d_1 (\dot{Z}_1 - \dot{Y} - l_1 \dot{\theta}) - K_2 (Z_2 - Y - l_2 \theta) - d_2 (\dot{Z}_1 - \dot{Y} - l_2 \dot{\theta}) \tag{3.16}$$

and $$q_b = - l_1 K_1 (Z_1 - Y - l_1 \theta) - l_1 d_1 (\dot{Z}_1 - \dot{Y} - l_2 \dot{\theta}) - l_2 K_2 (Z_2 - Y - l_2 \theta) - l_2 d_2 (\dot{Z}_2 - \dot{Y} - l_2 \dot{\theta}) \tag{3.17}$$

Substituting equations (3.9) and (3.10) in (3.8), the equation of motion for the front unsprung mass is obtained as :

$$M_1 \ddot{R}_1 = C_1 (X_1 - R_1) + s_1 (Y + L_1 \theta - R_1) + D_1 (\dot{Y} + L_1 \dot{\theta} - \dot{R}_1) \tag{3.18}$$

Similarly, for the rear unsprung mass,

$$M_2 \ddot{R}_2 = C_2 (X_2 - R_2) + s_2 (Y + L_2 \theta - R_2) + D_2 (\dot{Y} + L_2 \dot{\theta} - \dot{R}_2) \tag{3.19}$$

The equations for pitch and bounce motions are similarly derived. They are :

$$
\begin{aligned}
M \ddot{Y} = & - s_1 (Y + L_1 \theta - R_1) - D_1 (\dot{Y} + L_1 \dot{\theta} - \dot{R}_1) \\
& - s_2 (Y + L_2 \theta - R_2) - D_2 (\dot{Y} + L_2 \dot{\theta} - \dot{R}_2) \\
& - K_1 (Y + l_1 \theta - Z_1) - d_1 (\dot{Y} + l_1 \dot{\theta} - \dot{Z}_1) \\
& - K_2 (Y + l_2 \theta - Z_2) - d_2 (\dot{Y} + l_2 \dot{\theta} - \dot{Z}_2)
\end{aligned} \tag{3.20}
$$

and

$$
\begin{aligned}
J \ddot{\theta} = & - s_1 L_1 (Y + L_1 \theta - R_1) - D_1 L_1 (\dot{Y} + L_1 \dot{\theta} - \dot{R}_1) \\
& - s_2 L_2 (Y + L_2 \theta - R_2) - D_2 L_2 (\dot{Y} + L_2 \dot{\theta} - \dot{R}_2) \\
& - K_1 l_1 (Y + l_1 \theta - Z_1) - d_1 l_1 (\dot{Y} + l_1 \dot{\theta} - \dot{Z}_1) \\
& - K_2 l_2 (Y + l_2 \theta - Z_2) - d_2 l_2 (\dot{Y} + l_2 \dot{\theta} - \dot{Z}_2)
\end{aligned} \tag{3.21}
$$

3.4 DECOUPLING THE SYSTEM DIFFERENTIAL EQUATIONS

The governing differential equations derived in previous article consist of a finite number of system parameters which can be varied. If the number of degrees of freedom and number of system parameters can be reduced, the vibration isolation problem can be handled easily. This can be done by decoupling the system.

Laplace transform is applied to the basic differential equations (with zero initial conditions) and rearranged. A block diagram representation for the system is shown in figure (4) where,

$$G_1 = s_1 + s\, D_1 \tag{3.22}$$

$$G_2 = s_2 + s\, D_2 \tag{3.23}$$

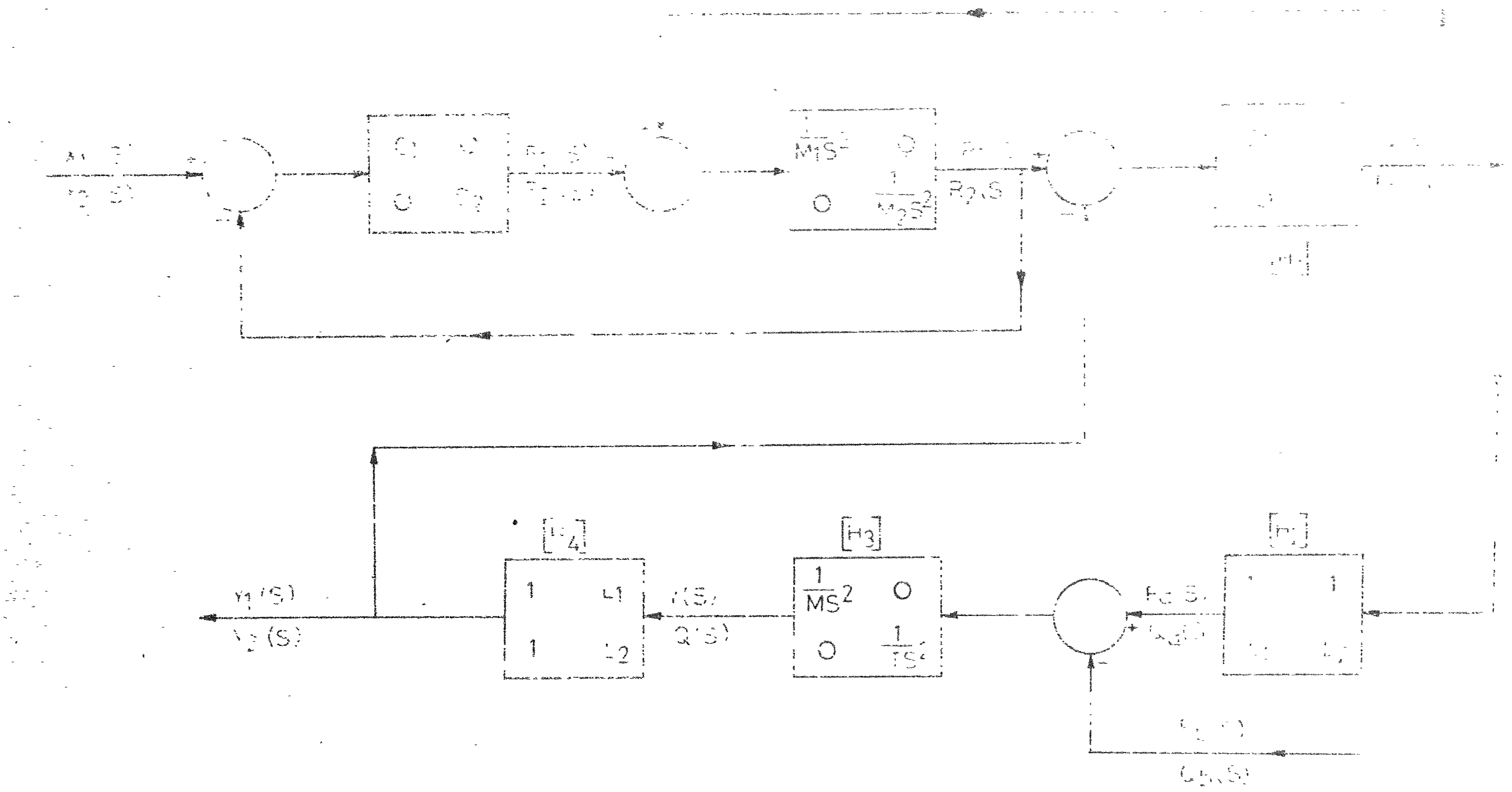

FIG. 4 BLOCK DIAGRAM FOR THE VEHICLE MODEL

Since not all the matrices are diagonal, the noninteraction condition can be satisfied if the product of the three matrices $[H_4]\ [H_3]\ [H_2]$ is diagonalized. That is :

$$\begin{bmatrix} \dfrac{1}{M\,s^2} + \dfrac{L_1^{\,2}}{J\,s^2} & \dfrac{1}{M\,s^2} + \dfrac{L_1\,L_2}{J\,s^2} \\[2ex] \dfrac{1}{M\,s^2} + \dfrac{L_1\,L_2}{J\,s^2} & \dfrac{1}{M\,s^2} + \dfrac{L_2^{\,2}}{J\,s^2} \end{bmatrix}$$ should be diagonal

Clearly, the condition for diagonalization is :

$$\frac{1}{M\,s^2} + \frac{L_1\,L_2}{J\,s^2} = 0 \qquad (3.24)$$

Substituting equation (3.3) in (3.24),

$$J = M\,ab \qquad (3.25)$$

Since $J = M\,K^2$, the front and rear suspensions will be independent with regards to their inputs and outputs if,

$$R = K^2 / ab = 1 \qquad (3.26)$$

This condition is known as "inertia decoupling", which permits the distributed mass M to be replaced by two point masses situated vertically above the axles. The two support points become reciprocal centresof percussion.

If, $R_1\ (s) = R_2\ (s) = 0$, then from the block diagram,

$$\begin{Bmatrix} Y \\ \theta \end{Bmatrix} = \Big[[I] + [H_3][H_2][H_1][H_4] \Big]^{-1} [H_3] \begin{Bmatrix} F_b \\ Q_b \end{Bmatrix} \qquad (3.27)$$

Since the matrices H_3 and I are diagonal, the condition for noninteraction of bounce and pitch motions of the body with

respect to the force f_b and moment q_b is that the product $[H_2][H_1][H_4]$ is diagonal. By actual multiplication the product of the three matrices is given by :

$$\begin{bmatrix} G_1 + G_2 & G_1 L_1 + G_2 L_2 \\ G_1 L_1 + G_2 L_2 & G_1 L_1^2 + G_2 L_2^2 \end{bmatrix}$$

In order that this matrix be diagonal :

$$G_1 L_1 + G_2 L_2 = 0 \tag{3.28}$$

Substituting equation (3.3) in (3.28),

$$a G_1 = b G_2 \tag{3.29}$$

Equation (3.29) implies,

$$\frac{S_1}{S_2} = \frac{D_1}{D_2} = \frac{b}{a} \tag{3.30}$$

Under these conditions, the force f_b excites only the bounce mode while the couple q_b excites only the pitch mode. This condition also ensures equal static deflection of the suspension at the front and the rear. The system is said to be completely noninteracting if equations (3.26) and (3.30) are satisfied simultaneously.

3.5 FREQUENCY RESPONSE FUNCTION FOR TWO DEGREE OF FREEDOM SYSTEMS

In the case where two masses are connected as in figure (5) to an oscillating foundation by linear springs and linear dashpots, the equations of motion in terms of absolute displacements are :

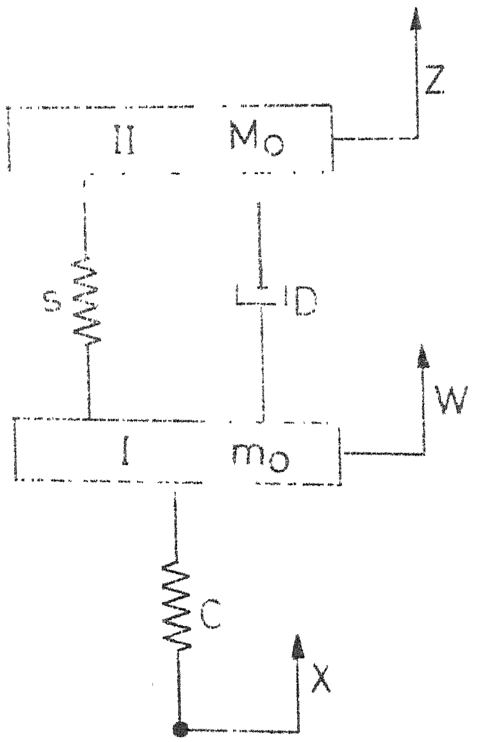

FIG. 5 TWO DEGREE OF FREEDOM SYSTEM

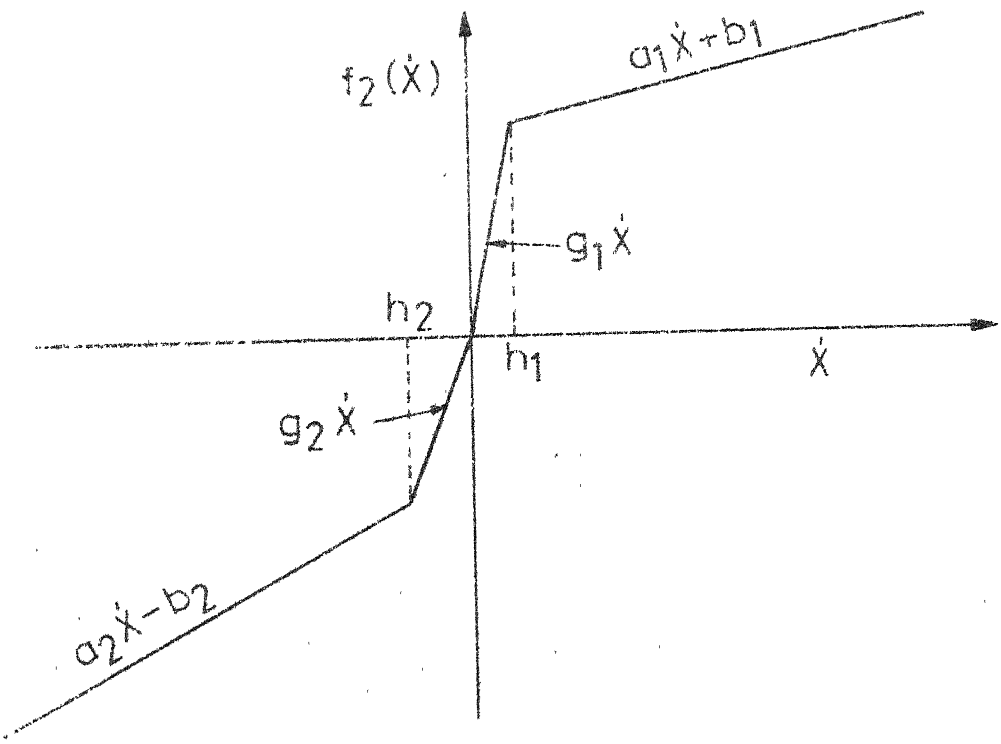

FIG. 6 DAMPING FORCE Vs VELOCITY ACROSS NONLINEAR DASHPOT

$$m_o \ddot{W} = -C(W - X) - s(W - Z) - D(\dot{W} - \dot{Z}) \quad (3.31)$$

and $$m_o \ddot{Z} = s(W - Z) + D(\dot{W} - \dot{Z}) \quad (3.32)$$

In order to standardize the subsequent treatment, the following notations are introduced

$$\omega_1 = \sqrt{C / m_o} \qquad \omega_2 = \sqrt{s / M_o}$$

$$\rho = M_o / m_o \qquad \xi_2 = D / 2\sqrt{s\ M_o} \quad (3.33)$$

The relative displacements between wheel and terrain (RW) and chassis and axle (RS) are,

$$RW = W - X$$

$$RS = Z - W \quad (3.34)$$

Substituting the equations (3.33) and (3.34) in the equations (3.31) and (3.32),

$$\ddot{RW} + \ddot{X} + \omega_1^2\ RW - \rho\,\omega_2^2\ RS - 2\rho\,\xi_2\omega_2\ \dot{RS} = 0 \quad (3.35)$$

and $$\ddot{RW} + \ddot{RS} + \ddot{X} + \omega_2^2\ RS + 2\,\xi_2\omega_2\ \dot{RS} = 0 \quad (3.36)$$

There are four different responses namely the two relative displacements defined in equation (3.34) and two absolute displacements of the two masses. The first pair is obtained by setting [17] ,

$$\ddot{X} = e^{i\omega t} \quad (3.37)$$

$$RW = H_{RW}\ e^{i\omega t} \quad (3.38)$$

and $$RS = H_{RS}\ e^{i\omega t} \quad (3.39)$$

in equations (3.35) and (3.36).

Where H_{RW} and H_{RS} are the complex frequency responses.

Solving these,

$$H_{RS}(\omega) = -\omega_1^2/\Delta \qquad (3.40)$$

Where $\Delta = \omega^4 - 2i(1+\rho)\xi_2\omega_2\omega^3$

$$-\omega^2\left[\omega_1^2 + (1+\rho)\omega_2^2\right] + 2i\xi_2\omega\omega_2\omega_1^2 + \omega_1^2\omega_2^2 \qquad (3.41)$$

Substituting equations (3.40) and (3.41) in (3.32), the frequency response function for the absolute acceleration $\ddot{Y}$ can be obtained as :

$$H_{\ddot{Y}} = (2i\xi_2\omega\omega_1^2\omega_2^2 + \omega_1^2\omega_2^2)/\Delta \qquad (3.42)$$

The absolute acceleration of the chassis and the displacement of chassis relative to the axles are of interest. Their frequency response functions are given by equations (3.40) and (3.42). The frequency response functions are useful in obtaining the power spectrum of the output if the input power spectrum is known.

3.6 FREQUENCY RESPONSE FUNCTION FOR LINEAR STRUCTURE WITH FINITELY MANY DEGREES OF FREEDOM

In the case of one or two degrees of freedom systems, the frequency response function between inputs such as displacement or acceleration and outputs such as absolute acceleration or relative displacement are simple and direct. But in the case of finitely many degrees of freedom systems it is not as simple. In dealing with the latter case it is convenient to use matrix notation. Therefore,

the dynamic characteristics of a linear finitely many degrees of freedom can be specified by a matrix of frequency response function $[H(\omega)]$.

To explain the physical meaning of a typical element $H_{jk}(\omega)$ in the matrix $[H(\omega)]$, the following is considered. Let a single sinusodial excitation act upon the system at a given point k and in a specified direction so that excitation is described as $f_k(t) = A \exp(i\omega t)$, where A is a complex coefficient. It is understood that the real part of the complex quantity only represents the physical input. The steady state response at point j has same frequency as the excitation $X_j(t) = A H_{jk}(\omega) \exp(i\omega t)$. Therefore, the frequency response function $H_{jk}(\omega)$ is the ratio of steady state response at point j to the sinusodial excitation at point k. In general this function is complex. For physical systems the elements in the matrix of frequency response function satisfy $H_{jk}(\omega) = H_{kj}(\omega)$.

The equations (3.14), (3.15) and (3.18) to (3.21) can be represented by a single matrix equation :

$$[m]\ddot{\overline{X}} + [C]\dot{\overline{X}} + [K]\overline{X} = \overline{f} \tag{3.43}$$

where $\overline{X} = \{R_1\ R_2\ Y\ \theta\ Z_1\ Z_2\}^T$

Since $C_1 = C_2$,

$$] = \frac{1}{C_1}\begin{bmatrix} M_1 & 0 & 0 & 0 & 0 & 0 \\ 0 & M_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & M & 0 & 0 & 0 \\ 0 & 0 & 0 & J & 0 & 0 \\ 0 & 0 & 0 & 0 & m_1 & 0 \\ 0 & 0 & 0 & 0 & 0 & m_2 \end{bmatrix}$$

$$] = \frac{1}{C_1}\begin{bmatrix} D_1 & 0 & -D_1 & -L_1D_1 & 0 & 0 \\ 0 & D_2 & -D_2 & -L_2D_2 & 0 & 0 \\ -D_1 & -D_2 & D_1+D_2+d_1+d_2 & L_1D_1+L_2D_2+l_1d_1+l_2d_2 & -d_1 & -d_2 \\ -L_1D_1 & -L_2D_2 & L_1D_1+L_2D_2+l_1d_1+l_2d_2 & L_1^2D_1+L_2^2D_2+l_1^2d_1+l_2^2d_2 & -l_1d_1 & -l_2d_2 \\ 0 & 0 & -d_1 & -l_1d_1 & d_1 & 0 \\ 0 & 0 & -d_2 & -l_2d_2 & 0 & d_2 \end{bmatrix}$$

$$] = \frac{1}{C_1}\begin{bmatrix} C_1+s_1 & 0 & -s_1 & -L_1s_1 & 0 & 0 \\ 0 & C_2+s_2 & -s_2 & -L_2s_2 & 0 & 0 \\ -s_1 & -s_2 & s_1+s_2+k_1+k_2 & L_1s_1+L_2s_2+l_1k_1+l_2k_2 & -k_1 & -k_2 \\ -L_1s_1 & -L_2s_2 & L_1s_1+L_2s_2+l_1k_1+l_2k_2 & L_1^2s_1+L_2^2s_2+l_1^2k_1+l_2^2k_2 & -k_1l_1 & -k_2l_2 \\ 0 & 0 & -k_1 & -k_1l_1 & k_1 & 0 \\ 0 & 0 & -k_2 & -k_2l_2 & 0 & k_2 \end{bmatrix}$$

$$\overline{f} = \left\{ X_1 \quad X_2 \quad 0 \quad 0 \quad 0 \quad 0 \right\}^T$$

The matrix of frequency response functions, $[H(\omega)]$ can be obtained by letting the excitation vector $\bar{f} = \bar{f}_o \exp(i\omega t)$, where $\bar{f}_o$ is a vector of constants. Let the response vector be,

$$\bar{X} = [H(\omega)]\ \bar{f}. \text{ Thus,}$$

$$[H(\omega)] = \left[-\omega^2 [m] + i\omega [C] + [k]\right]^{-1} \tag{3.44}$$

Since $[m]$, $[C]$ and $[k]$ are 6 x 6 matrices, $[H(\omega)]$ is also a 6 x 6 matrix.

3.7 METHOD OF EQUIVALENT LINEARIZATION

The damping coefficients and spring constants considered in section (3.3) and (3.6) are nonlinear as is shown in figures (7) to (11). The method of equivalent linearization has been used as follows.

The differential equation of motion, for nonlinear oscillator subject to stationary Gaussian excitation, is given by,

$$\ddot{X} + g(X, \dot{X}, t) = f(t) \tag{3.45}$$

where $g(X, \dot{X}, t)$ is a nonlinear function. Further $g(X, \dot{X}, t)$ is assumed to be weakly nonlinear [15].

Rewriting equation (3.45),

$$\ddot{X} + S\dot{X} + D_X + e(X, \dot{X}, t) = f(t) \tag{3.46}$$

where D is the"equivalent linear damping" coefficient per unit mass, S is the "equivalent linear stiffness" coefficient per unit mass and $e(X, \dot{X}, t)$ is the error due to linearization.

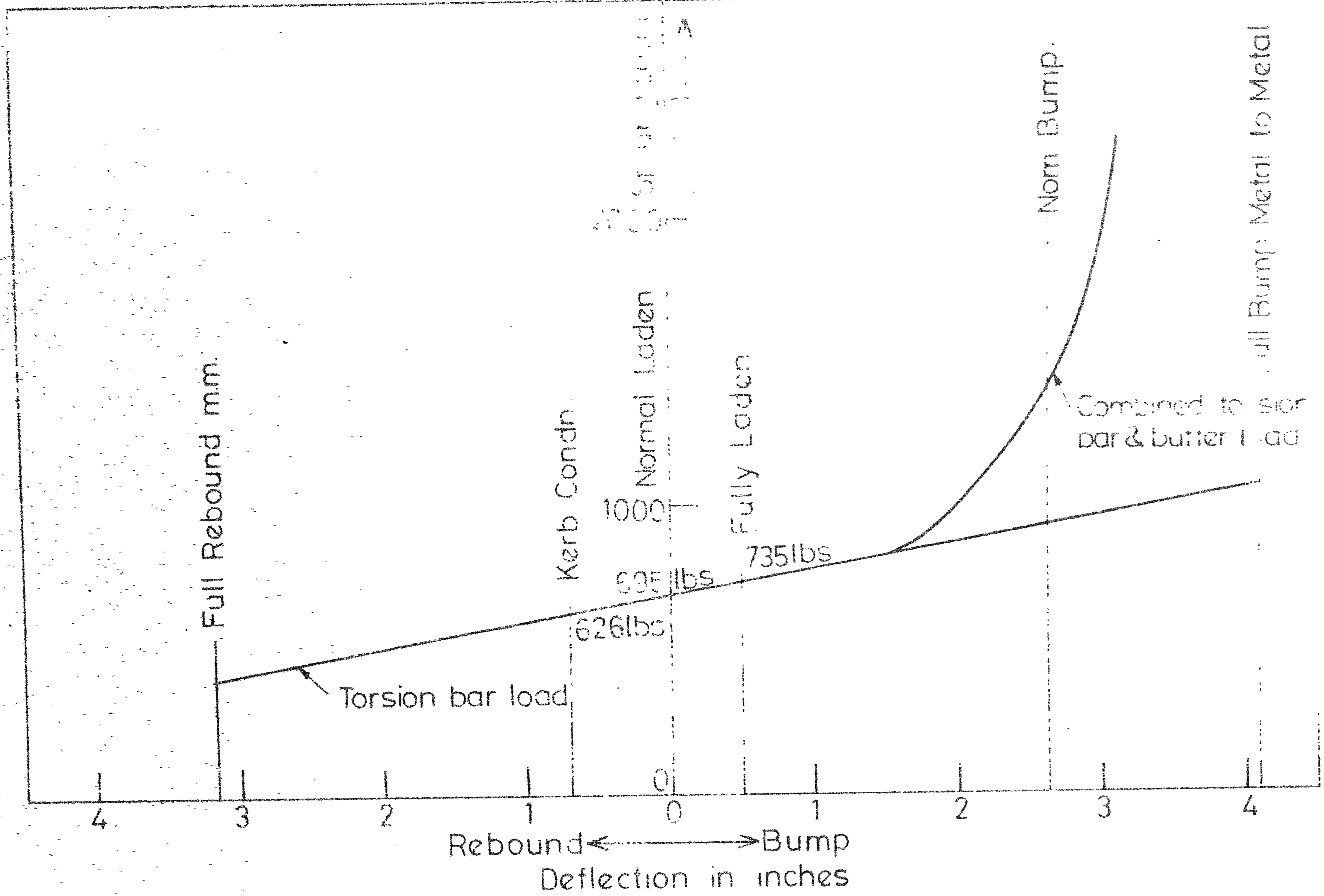

FIG.7 LOAD-DEFLECTION CURVE OF A PASS. CAR. FRONT SUSPN. & BUFFER

(Courtsey: Hindustan Motors Ltd. Calcutta)

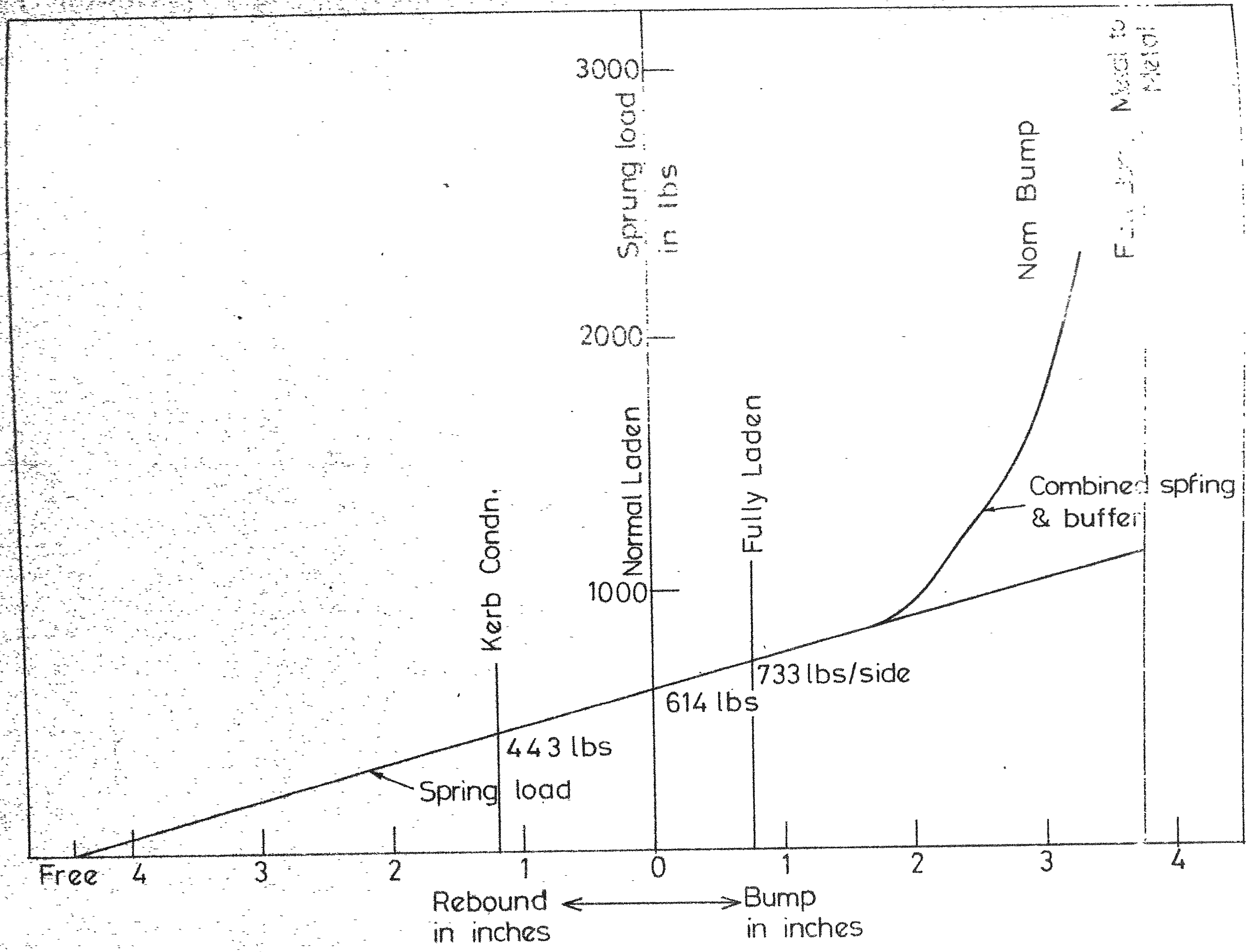

FIG.8 LOAD DEFLECTION CURVE OF A PASS. CAR. REAR SPRING & BUFFER

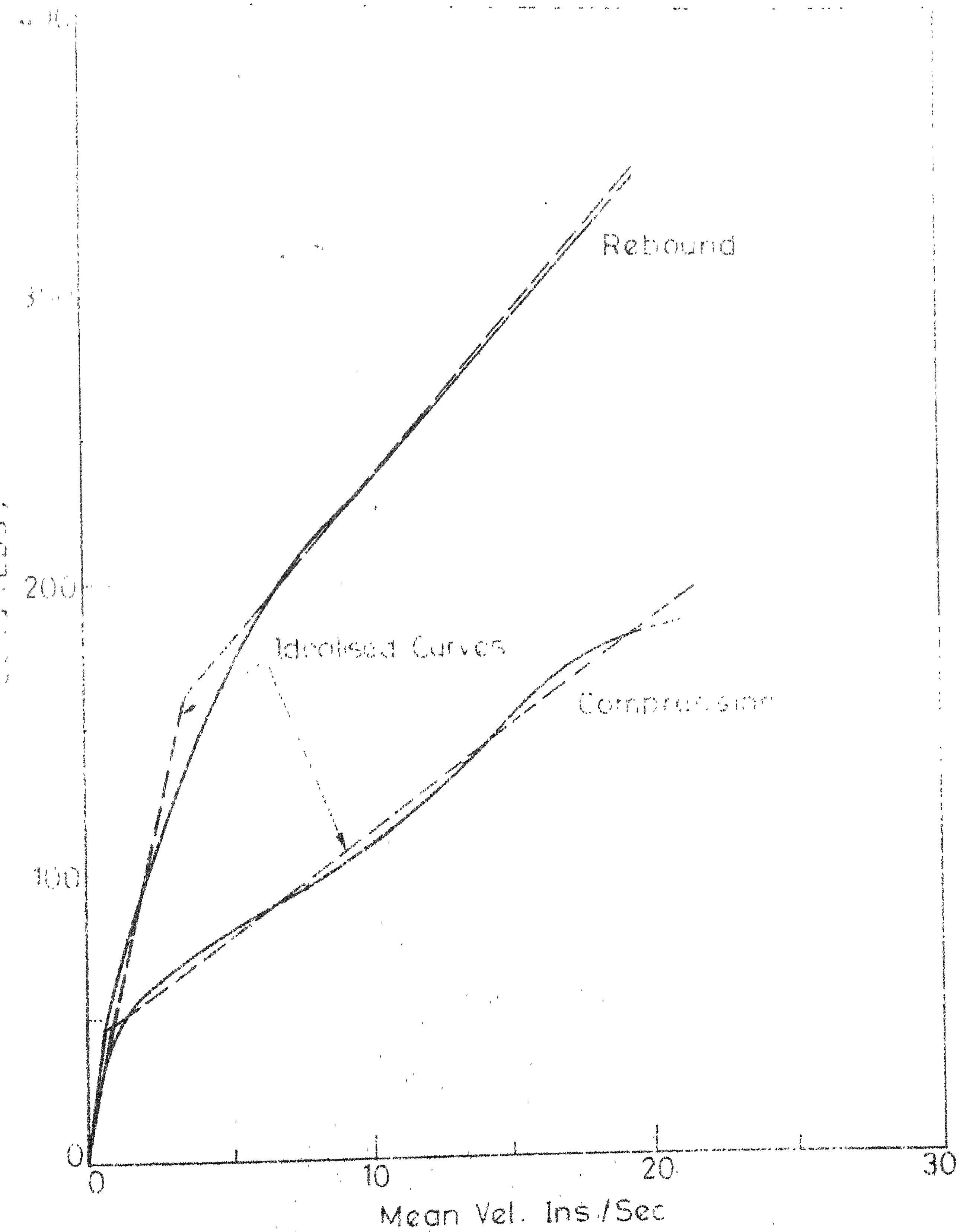

FIG. 9 VELOCITY DAMPING FORCE CURVE FOR HYDRAULIC MAKE FRONT S/A (AT 7.1441)
(Courtsey: [illegible] Motors [illegible], Calcutta)

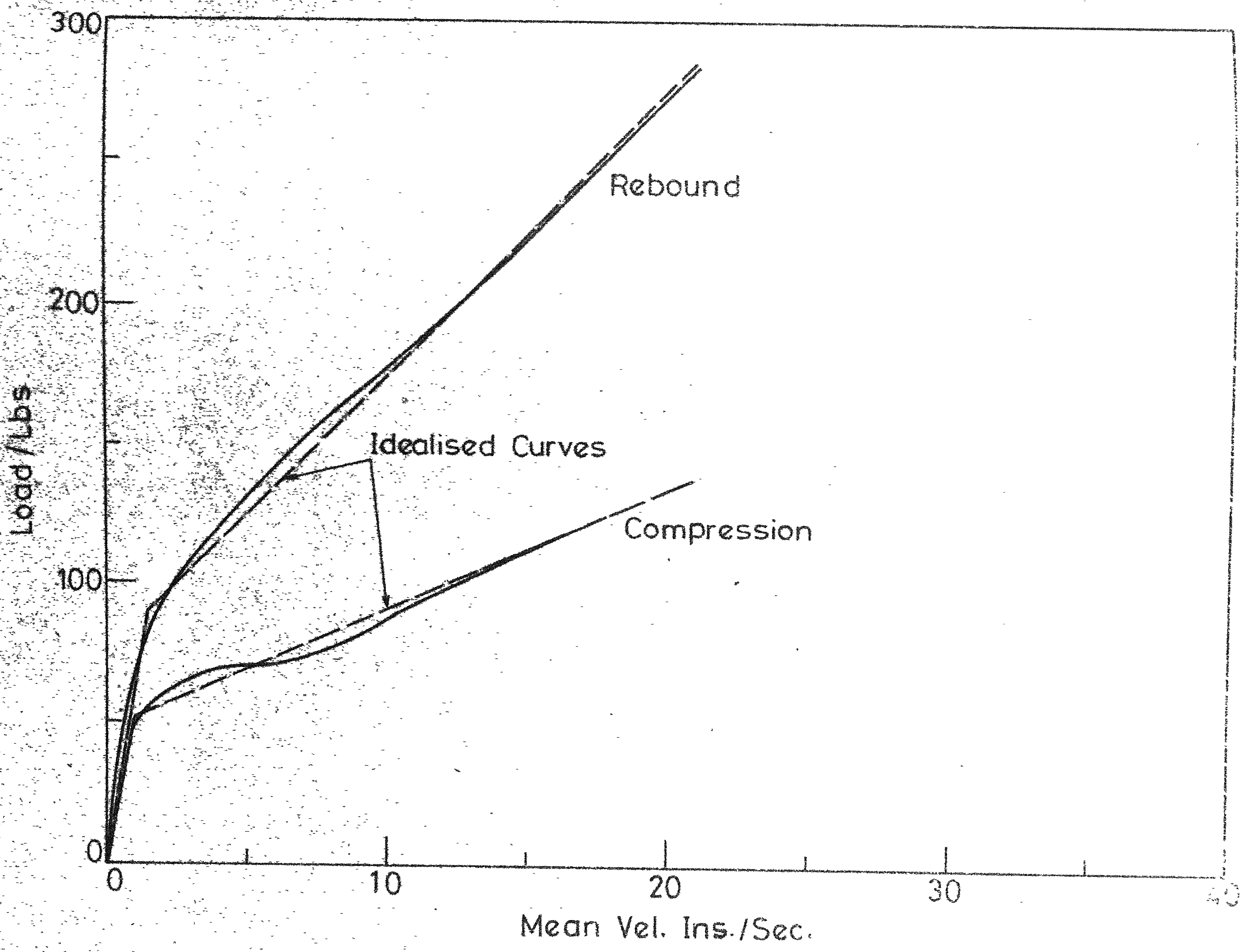

FIG. 10 VELOCITY DAMPING FORCE CURVE FOR HYDRAULICS MAKE REAR S/A (AT 7.1442) (Courtsey : Hindustan Motors Ltd. Calcutta)

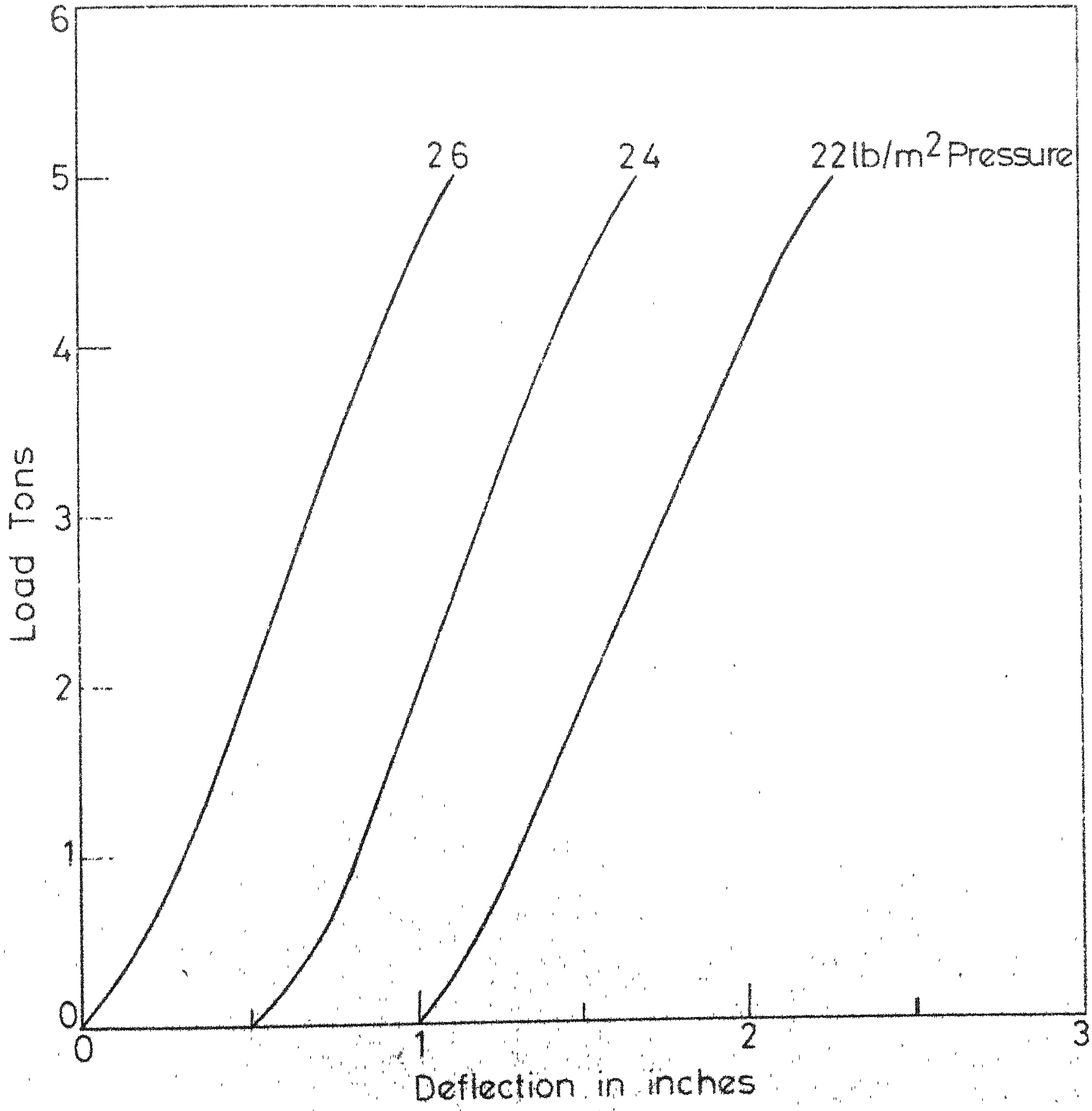

FIG. 11 STATIC LOAD-DEFLN. CHARACTERISTICS OF TIRES.
TIRE SIZE :- 5.90 15, C 49
(Courtsey : Hindustan Motors Ltd. Calcutta).

If $e\ (X, \dot{X}, t)$ is neglected, equation (3.46) becomes linear and may be readily solved. The logical choice of S and D is therefore, those values that make $e\ (X, \dot{X}, t)$ a minimum. One way of achieving this is to minimize the expected mean-squared-error.

Comparing equations (3.45) and (3.46),

$$e\ (X, \dot{X}, t) = g\ (X, \dot{X}, t) - D\ \dot{X} - S_X \tag{3.47}$$

Now minimizing the expectation of the mean squared error $E\left[e^2\ (X, \dot{X}, t)\right]$ with respect to D and S and interchanging the order of differentiation and expectation,

$$E\left[\dot{X}\, g\ (X, \dot{X}, t)\right] - D\, E\left[\dot{X}^2\right] - S\, E\left[X\, \dot{X}\right] = 0$$

and $$E\left[X\, g\ (X, \dot{X}, t)\right] - D\, E\left[X\, \dot{X}\right] - S\ E\left[X^2\right] = 0 \tag{3.48}$$

A stationary random process is orthogonal to its derivative [14] that is,

$$E\left[X\ \dot{X}\right] = 0 \tag{3.49}$$

Hence solving for D and S,

$$D = E\left[\dot{X}\, g\ (X, \dot{X}, t)\right] / E\left[\dot{X}^2\right]$$

and $$S = E\left[X\, g\ (X, \dot{X}, t)\right] / E\left[X^2\right] \tag{3.50}$$

If the nonlinearities in stiffness and damping are linearly related, $g\ (X, \dot{X}, t)$ can be expressed as,

$$g\ (X, \dot{X}, t) = f_1\ (X) + f_2\ (\dot{X}) \tag{3.51}$$

Since, for a Gaussian process equation (3.49) implies independence of X and $\dot{X}$, from equation (3.50) and (3.51),

$$D = E\left[\dot{X} \; f_2(\dot{X})\right] / E\left[\dot{X}^2\right] \tag{3.52}$$

and
$$S = E\left[X \; f_1(X)\right] / E\left[X^2\right] \tag{3.53}$$

From the figures (7) and (8) it is observed that, in the range of relative displacement between axles and suspension, the spring characterstics are linear, that is, $f_1(X) = k_X$. Substituting this value in equation (3.53), we obtain $S = k$. To determine the equivalent linear damping coefficient D equation (3.52) will be used.

Referring to figure (6), the nonlinear [8] $f_2(\dot{X})$ is given by,

$$f_2(\dot{X}) = \begin{cases} a_2 \dot{X} - b_2 & -\infty \leq \dot{X} \leq -h_2 \\ g_2 \dot{X} & -h_2 \leq \dot{X} \leq 0 \\ g_1 \dot{X} & 0 \leq \dot{X} \leq h_1 \\ a_1 \dot{X} + b_1 & h_1 \leq \dot{X} \leq \infty \end{cases} \tag{3.54}$$

Equation (3.52) can be written as,

$$D = \frac{\int_{-\infty}^{\infty} \dot{X} f_2(\dot{X}) \, p(\dot{X}) \, d\dot{X}}{\int_{-\infty}^{\infty} \dot{X}^2 \, p(\dot{X}) \, d\dot{X}} \tag{3.55}$$

Where $p(\dot{X})$ is the probability density function. Since $\dot{X}$ is a Gaussian random variable (zero mean and variance σ^2) $p(\dot{X})$ is given by

$$p(\dot{X}) = \frac{1}{\sqrt{2\pi}\,\sigma} e^{-\dot{X}^2/2\sigma^2} \tag{3.56}$$

Substituting equation (3.54) in (3.55),

$$D = \left[\int_{-\infty}^{-h_2} \dot{X} (a_2 \dot{X} - b_2)\, p(\dot{X})\, d\dot{X} + \int_{-h_2}^{0} \dot{X} (g_2 \dot{X})\, p(\dot{X})\, d\dot{X} + \int_{0}^{h_1} \dot{X} (g_1 \dot{X})\, p(\dot{X})\, d\dot{X} + \int_{h_1}^{\infty} (a_1 \dot{X} + b_1) \dot{X}\, p(\dot{X})\, d\dot{X} \right] / \sigma^2 \tag{3.57}$$

Since $p(\dot{X})$ is an even function,

$$D = \frac{a_1}{\sigma^2} \int_{h_1}^{\infty} \dot{X}^2 p(\dot{X})\, d\dot{X} + \frac{b_1}{\sigma^2} \int_{h_1}^{\infty} \dot{X}\, p(\dot{X})\, d\dot{X} + \frac{g_1}{\sigma^2} \int_{0}^{h_1} \dot{X}^2 p(\dot{X})\, d\dot{X} + \frac{a_2}{\sigma^2} \int_{h_2}^{\infty} \dot{X}^2 p(\dot{X})\, p(\dot{X})\, d\dot{X} + \frac{b_2}{\sigma^2} \int_{h_2}^{\infty} \dot{X}\, p(\dot{X})\, p(\dot{X})\, d\dot{X} + \frac{g_2}{\sigma^2} \int_{0}^{h_2} \dot{X}^2 p(\dot{X})\, d\dot{X} \tag{3.58}$$

Let $I_1 = \int_{h}^{\infty} \dot{X}\, p(\dot{X})\, d\dot{X} = \frac{1}{\sqrt{2\pi}\,\sigma} \int_{h}^{\infty} \dot{X}\, e^{-\dot{X}^2/2\sigma^2}\, d\dot{X}$

Putting $Y = \dot{X}^2 / 2\sigma^2$,

$$I_1 = \frac{\quad}{\sqrt{2\pi}}\, e^{-h^2/2\sigma^2} \tag{3.59}$$

Let $I_2 = \int_{0}^{h} \frac{\dot{X}^2\, e^{-X^2/2\sigma^2}\, d\dot{X}}{\sqrt{2\pi}\,\sigma}$

Putting $Y = \dot{X} / \sqrt{2}\,\sigma$,

$$I_2 = \frac{2\sigma^2}{\quad} \int^{h/\sqrt{2}\,\sigma} Y^2\, e^{-y^2}\, dy .$$

Let the following integral be considered,

$$\int e^{-y^2} \, dy$$

If $u = e^{-y^2}$ and $v = y$, then,

$$\int e^{-y^2} \, dy = y \, e^{-y^2} + \int 2 \, y^2 \, e^{-y^2} \, dy$$

$$\text{or} \int y^2 \, e^{-y^2} \, dy = \frac{1}{2} \int e^{-y^2} \, dy - \frac{1}{2} \, y \, e^{-y^2}$$

$$\text{Therefore,} \quad I_2 = - \frac{\sigma^2}{\sqrt{\pi}} \left[y \, e^{-y^2} \right]_0^{h/\sqrt{2}\,\sigma} + \frac{\sigma^2}{\sqrt{\pi}} \int_0^{h/\sqrt{2}\,\sigma} e^{-y^2} \, dy$$

$$= \frac{\sigma^2}{\sqrt{\pi}} \times \frac{\sqrt{\pi}}{2} \left(\frac{2}{\sqrt{\pi}} \int_0^{h/\sqrt{2}\,\sigma} e^{-y^2} \, dy \right) - \frac{h\sigma}{\sqrt{2\pi}} \, e^{-h^2/2\sigma^2}$$

$$= \frac{\sigma^2}{2} \, \text{erf} \left(\frac{h}{\sqrt{2}\,\sigma} \right) - \frac{h\sigma}{\sqrt{2\pi}} \, e^{-h^2/2\sigma^2} \tag{3.60}$$

Similarly,

$$I_3 = \int_h^{\infty} \frac{\dot{X}^2 \, e^{-\dot{X}/2\sigma^2} \, d\dot{X}}{\sqrt{2\pi}\,\sigma}$$

$$= \frac{\sigma^2}{2} \, \text{erfc} \left(\frac{h}{\sqrt{2}\,\sigma} \right) + \frac{h\sigma}{\sqrt{2\pi}} \, e^{-h^2/2\sigma^2} \tag{3.61}$$

Substituting equations (3.59), (3.60) and (3.61) in (3.58),

$$\begin{aligned} D = {} & \frac{e^{-h_1^2/2\sigma^2}}{\sqrt{2\pi}\,\sigma} \left(b_1 + a_1 h_1 - g_1 h_1 \right) \\ & + \frac{e^{-h_2^2/2\sigma^2}}{\sqrt{2\pi}\,\sigma} \left(b_2 + a_2 h_2 - g_2 h_2 \right) \\ & + \frac{a_1}{2} \, \text{erfc} \left(\frac{h_1}{\sqrt{2}\,\sigma} \right) + \frac{a_2}{2} \, \text{erfc} \left(\frac{h_2}{\sqrt{2}\,\sigma} \right) \\ & + \frac{g_1}{2} \, \text{erf} \left(\frac{h_1}{\sqrt{2}\,\sigma} \right) + \frac{g_2}{2} \, \text{erf} \left(\frac{h_2}{\sqrt{2}\,\sigma} \right) \end{aligned} \tag{3.62}$$

Considering the special case, when the nonlinear term consists of Coulomb friction and linear viscous damping, that is, putting $h_1 = h_2 = 0$ in equation (3.62), the expression for D becomes,

$$D = \frac{a_1 + a_2}{2} + \frac{b_1 + b_2}{\sqrt{2\pi}\,\sigma} \tag{3.63}$$

3.8 DETERMINATION OF VARIANCE OF VELOCITY ACROSS THE DASHPOT

The variance of the velocity of sprung mass relative to unsprung mass $\sigma_{\dot{X}}^2$ is needed in order to determine the equivalent linear damping of the suspension dashpot. The velocity of sprung mass relative to axle is given by,

$$\dot{X} = \dot{Z} - \dot{W} \tag{3.64}$$

Where, $\dot{Z} = \dot{Y} + L\,\dot{\theta}$ (3.65)

The two terms $\dot{Y}$ and $L\,\dot{\theta}$ on the right hand side of equation (3.65) are random variables with zero mean and variance $\sigma_{\dot{Y}}^2$ and $L^2\,\sigma_{\dot{\theta}}^2$ respectively. Therefore $\dot{Z}$ is also a "trend free" random variable and variance of $\dot{Z}$ is given by [27],

$$\sigma_{\dot{Z}}^2 = \sigma_{\dot{Y}}^2 + L^2\,\sigma_{\dot{\theta}}^2 + 2\,\rho_1\,L\,\sigma_{\dot{Y}}\,\sigma_{\dot{\theta}} \tag{3.66}$$

where ρ_1 is correlation coefficient between $\dot{Y}$ and $\dot{\theta}$. Similarly the mean value of $\dot{X}$ is zero and variance of $\dot{X}$ is given by,

$$\sigma_{\dot{X}}^2 = \sigma_{\dot{Z}}^2 + \sigma_{\dot{W}}^2 - 2\,\rho_2\,\sigma_{\dot{Z}}\,\sigma_{\dot{W}} \tag{3.67}$$

where ρ_2 is the correlation between $\dot{Z}$ and $\dot{W}$. ρ_2 can be calculated from the following equation, [23]

$$\rho_2 = \frac{R_{\dot{W}\dot{Z}}(0)}{\sqrt{R_{\dot{W}}(0)\;R_{\dot{Z}}(0)}} \tag{3.68}$$

where $R_{\dot{W}\dot{Z}}(0)$ is the cross-correlation function between the random variables $\dot{W}$ and $\dot{Z}$. $R_{\dot{W}}(0)$, $R_{\dot{Z}}(0)$ and $R_{\dot{W}\dot{Z}}(0)$ can be calculated from matrix of cross spectral densities. Applying equation (2.6),

$$R_{\dot{W}}(0) = \int_{-\infty}^{\infty} S_{\dot{W}}(\omega)\,d\omega$$

$$\text{and} \quad R_{\dot{Z}}(0) = \int_{-\infty}^{\infty} S_{\dot{Z}}(\omega)\,d\omega \tag{3.69}$$

Similarly,

$$R_{\dot{W}\dot{Z}}(0) = \int_{-\infty}^{\infty} S_{\dot{W}\dot{Z}}(\omega)\,d\omega \tag{3.70}$$

where $S_{\dot{W}\dot{Z}}(\omega)$ is the cross spectral density function between $\dot{Y}$ and $\dot{Z}$.

CHAPTER IV

RESPONSE OF THE VEHICLE TO ROAD ROUGHNESS

4.1 INTRODUCTION

As an automobile runs on a road, it is subject to random excitation. The responses, such as vertical displacement and acceleration of different components of the automobile and its passengers, are of interest to the automotive engineer. In this chapter, the response of the automobile has been investigated. A mathematical model of the vehicle is proposed in the last chapter. For a stationary road input and a linear system, the response can be determined from the complex frequency response and the input power spectrum. But for a nonlinear system, the response is difficult to determine. However, for a weakly nonlinear system, an equivalent linear system can be found out, which in effect should give approximately the same response, as would a nonlinear system. For such a linear system, the complex frequency response is a function of the input. Response quantities sought in this investigation are the displacement of sprung mass relative to the axles, absolute acceleration of the seats and riders. It is also intended to study the effects of various parameters of the system on different responses and to select such a set of parameters that would give maximum comfort to riders. The method to determine various responses is described in the following sections.

4.2 RESPONSE OF A MULTI-DEGREE OF FREEDOM SYSTEM

The set of inputs to the system are represented by a vector $\overline{F}(t)$ and outputs by $\overline{X}(t)$. The relation between input and output is given by,

$$\overline{X}(t) = \int_0^t \left[h(t-\tau)\right] \overline{F}(\tau)\, d\tau \tag{4.1}$$

where the integration is interpreted in the sense of mean square convergence and $\left[h(t)\right]$ is the matrix of impulse response functions. A typical element $h_{jk}(t)$ in the matrix of impulse response function, describes the motion at node j due to a unit impulse excitation at node k applied at time $t = 0$. The matrix of impulse response function and the matrix of complex frequency response form a Fourier transform pair .,

$$\left[h(t)\right] = \frac{1}{2\pi} \int_{-\infty}^{\infty} \left[H(\omega)\right] \exp(i\omega t)\, d\omega \tag{4.2}$$

$$\left[H(\omega)\right] = \int_{-\infty}^{\infty} \left[h(t)\right] \exp(-i\omega t)\, dt \tag{4.3}$$

In the theory of random vibration, the first and second order statistical properties are of importance. They are determined from the relation (4.1). As operators expectation and integration are linear,

$$E\left[\overline{X}(t)\right] = \int_0^t \left[h(t-\tau)\right] E\left\{\overline{F}(\tau)\right\} d\tau \tag{4.4}$$

$$E\left[\overline{X}(t_1)\,\overline{X}'(t_2)\right] = \int_0^{t_1}\int_0^{t_2} \left[h(t_1-\tau_1)\right] E\left\{\overline{F}(\tau_1)\,\overline{F}'(\tau_2)\right\} \left[h'(t_2-\tau_2)\right] d\tau_1\, d\tau_2 \tag{4.5}$$

The excitation is "trend free". Therefore, $E\left[\overline{X}(t)\right]$ is a null vector, as can be readily seen from equation (4.4). For a weakly stationary excitation, the correlation matrix of the excitation vector, depends only on the time difference. The matrix is denoted by

$$\left[R_{\overline{F}}(\tau_1 - \tau_2)\right] = E\left\{\overline{F}(\tau_1)\,\overline{F}'(\tau_2)\right\} \tag{4.6}$$

If it is assumed that every element in matrix $R_{\overline{F}}$ is continuous, absolutely integrable and of bounded variation in the entire domain of the argument, then $\left[R_{\overline{F}}(\tau_1 - \tau_2)\right]$ can be Fourier transformed:

$$\left[R_{\overline{F}}(\tau_1 - \tau_2)\right] = \int_{-\infty}^{\infty}\left[\phi_{\overline{F}}(\omega)\right] \exp\left\{ i\omega(\tau_1 - \tau_2)\right\} d\omega \tag{4.7}$$

and $$\left[\phi_{\overline{F}}(\omega)\right] = \frac{1}{2\pi}\int_{-\infty}^{\infty}\left[R_{\overline{F}}(u)\right] \exp(-i\omega u)\, du \tag{4.8}$$

where $\left[\phi_{\overline{F}}(\omega)\right]$ is the matrix of cross spectral densities of the excitations. Substituting equation (4.7) in equation (4.5) and interchanging the order of integration,

$$E\left[\overline{X}(t_1)\,\overline{X}'(t_2)\right] = \int_{-\infty}^{\infty}\left[H(\omega, t_1)\right]\left[\phi_{\overline{F}}(\omega)\right]\left[H^*(\omega, t_2)\right] \exp\left\{ i\omega(t_1 - t_2)\right\} d\omega \tag{4.9}$$

where $$\left[H(\omega, t)\right] = \int_{0}^{t}\left[h(u)\right] \exp(-i\omega u)\, du$$

$$= \int_{-\infty}^{t}\left[h(u)\right] \exp(-i\omega u)\, du \tag{4.10}$$

Since the excitation begins at $t = 0$, $\left[h(u)\right]$ is a null matrix for $t < 0$. If t_1 and t_2 tend to infinity, keeping the difference $t_1 - t_2$ constant, equation (4.9) yields correlation matrix of the response,

$$R_{\overline{X}}(t_1 - t_2) = \int_{-\infty}^{\infty} \left[H(\omega)\right] \left[\phi_{\overline{F}}(\omega)\right] \left[H^*(\omega)\right] \exp\left\{i\omega(t_1 - t_2)\right\} d\omega \qquad (4.11)$$

Comparing equation (4.11) with (2.5),

$$\left[\phi_{\overline{X}}(\omega)\right] = \left[H(\omega)\right]\left[\phi_{\overline{F}}(\omega)\right]\left[H^*(\omega)\right] \qquad (4.12)$$

For a single degree of freedom system equation (4.12) reduces to,

$$\phi_X(\omega) = \left|H(\omega)\right|^2 \phi_F(\omega) \qquad (4.13)$$

where $\left|H(\omega)\right|^2$ is known as transmittancy function.

4.3 RESPONSE OF TWO-DEGREE OF FREEDOM SYSTEM

Let X (t) and $\ddot{X}$ (t) be the displacement and acceleration input to the two degree of freedom system. The corresponding one sided power spectrum $S_X(\omega)$ and $S_{\ddot{X}}(\omega)$ are defined in the range ω_1 to ω_2. For a stationary input the two power spectrum are related:

$$S_{\ddot{X}}(\omega) = \omega^4 S_X(\omega) \qquad \text{for } \omega_1 \leq \omega \leq \omega_2 \qquad (2.27)$$

Let $\ddot{Y}$ (t) be the absolute acceleration of the mass II (figure 5). The frequency response function for $\ddot{X}$ (t) as input and $\ddot{Y}$ (t) as output are derived earlier.

$$H_{\ddot{Y}} = \frac{i\,2\,\xi_2\omega\,\omega_1^2\,\omega_2^2 + \omega_1^2\,\omega_2^2}{\Delta} \qquad (3.42)$$

where, $$\Delta = \omega^4 - i\,2\,(1+\rho)\,\xi_2\,\omega_2\,\omega^3 - \omega^2\left[\omega_1^2 + (1+\rho)\,\omega_2^2\right] + i\,2\,\xi_2\,\omega\,\omega_2\,\omega_1^2 + \omega_1^2\omega_2^2 \qquad (3.41)$$

To get an input-output power spectrum relationship, equation (4.13) is applied :

$$S_{\ddot{Y}}(\omega) = \left| H_{\ddot{Y}}(\omega) \right|^2 S_{\ddot{X}}(\omega) \tag{4.14}$$

As the input to the linear system is "trend free", the output also has zero mean. The variance can be determined as,

$$E\left[\ddot{Y}^2\right] = \int_{\omega_1}^{\omega_2} \left| H_{\ddot{Y}}(\omega) \right|^2 S_{\ddot{X}}(\omega)\, d\omega \tag{4.15}$$

In order to nondimensionalize, equation (4.15) is divided by $E\left[\ddot{X}^2\right]$,

$$\frac{E\left[\ddot{Y}^2\right]}{E\left[\ddot{X}^2\right]} = \frac{\int_{\omega_1}^{\omega_2} \frac{N_{\ddot{Y}}}{D} S_{\ddot{X}}(\omega)\, d\omega}{\int_{\omega_1}^{\omega_2} S_{\ddot{X}}(\omega)\, d\omega} \tag{4.16}$$

where,

$$N_{\ddot{Y}} = \omega_1^4 \omega_2^4 + 4 \zeta_2^2 \omega^2 \omega_1^4 \omega_2^4 \tag{4.17}$$

and

$$D = \left[\omega^4 - \omega^2\left\{\omega_1^2 + (1+f)\omega_2^2\right\} + \omega_1^2\omega_2^2\right]^2 + \left[-2\omega^3(1+f)\zeta_2\omega_2 + 2\zeta_2\omega\omega_2\omega_1^2\right]^2 \tag{4.18}$$

Similarly, the ratio of variance of relative displacement of mass II with respect to mass I and variance of displacement input can similarly be derived :

$$\frac{E[RS^2]}{E[X^2]} = \frac{\int_{\omega_1}^{\omega_2} |H_{RS}|^2 \, S_{\ddot{X}}(\omega)\, d\omega}{\int_{\omega_1}^{\omega_2} S_X(\omega)\, d\omega}$$

$$\frac{\int_{\omega_1}^{\omega_2} \frac{N_{RS}}{D} \, S_{\ddot{X}}(\omega)\, d\omega}{\int_{\omega_1}^{\omega_2} S_X(\omega)\, d\omega} \tag{4.19}$$

where $N_{RS} = \omega_1^4$ (4.20)

4.4 RIDE COMFORT EVALUATION

The response of the seats inside the vehicle acts as the input to the passenger. The input vibration is described in terms of power spectrum. Response of human being to random vibration input has not been studied much. However in the low frequency range response of persons to sinusoidal inputs has been extensively studied. Results obtained by various investigators differ from each other. This may be due to variations in physiological reactions in individuals as well as fatigue (physical as well as mental) level of test subjects at the time of conducting the experiment. In this investigation, the recommendation offered by the International Standard Organisation (I.S.O.) is considered. (figure 12)

Recently it has been investigated whether the evaluation system applicable only to simple sinusoidal vibration, could also be used with well founded validity for mixtures of sinusoidal vibrations,

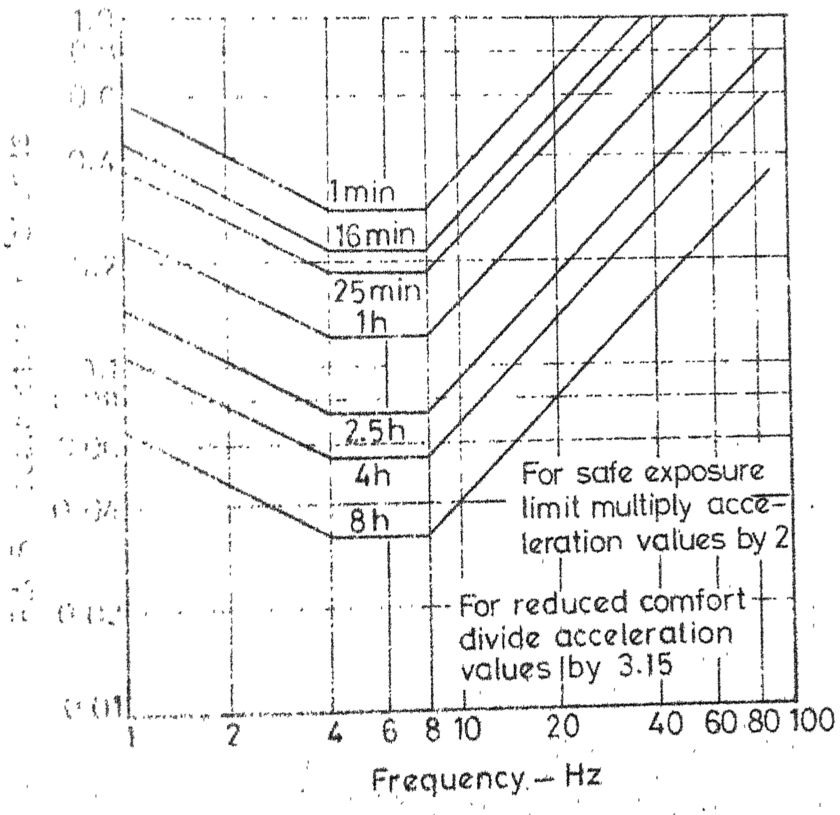

FIG. 12 I S O PROPOSED RECOMMENDATIONS FOR THE EFFECTS OF VERTICAL VIBRATION ON MAN

and for random vibrations imposed on seated persons [28]. With reference to tolerance limit it has been observed that formation of a general effective value from the square root of the sum of squares of the weighted acceleration amplitudes of two vibration components does not exactly correspond to the subjective physiological effect.

In this investigation the man is assumed to be a linear system, having a transfer function defined as the ratio of ride sensation to input spectrum. The transfer function T (f) is a function of frequency f, and is given by

$$T\ (f) = \frac{ZZ\ (f)}{Z\ \ (f)} \tag{4.21}$$

where, ZZ (f) is ride sensation

Z (f) is input to the rider.

Let C (f) be the numerical value of the r.m.s. acceleration, a human being can tolerate (figure 12, 8 hour limit curve). The transfer function T (f) can be determined from the following relation,

$$T\ (f) = \frac{C\ (4)}{C\ (f)} \tag{4.22}$$

where, C (4) is the numerical value of r.m.s, acceleration at frequency of 4 Hz. From equations (4.21) and (4.22) equivalent ride sensation at frequency of 4 Hz can be determined. If G (f) is the input spectrum to the rider, the output variance can be determined from the following integral,

$$Var = \int_{f_1}^{f_2} G\ (f)\ C\ (f)\ df \tag{4.23}$$

4.5 DETERMINATION OF RESPONSES BY NUMERICAL INTEGRATION OF SYSTEM DIFFERENTIAL EQUATIONS

In Section 4.2, a method is described in which the responses are obtained using frequency response function. Instead of non-linear parameters, equivalent linear system parameters are used in the calculations. The validity of calculation in frequency domain analysis can be checked by an alternate approach of determining the response of the nonlinear system by direct numerical integration of the system differential equations.

The system is described by 6 second order differential equations. From these second order equations, 12 first order differential equations are obtained using the following notations :

$$Y_1\ (t) = R_1\ (t)$$
$$Y_2\ (t) = R_2\ (t)$$
$$Y_3\ (t) = Y\ (t)$$
$$Y_4\ (t) = \theta\ (t)$$
$$Y_5\ (t) = Z_1\ (t)$$
$$Y_6\ (t) = Z_2\ (t)$$
$$Y_7\ (t) = \dot{R}_1\ (t)$$
$$Y_8\ (t) = \dot{R}_2\ (t)$$
$$Y_9\ (t) = \dot{Y}\ (t)$$
$$Y_{10}(t) = \dot{\theta}\ (t)$$
$$Y_{11}(t) = \dot{Z}_1(t)$$

and $$Y_{12}(t) = \dot{Z}_2\ (t)$$

Substituting equation (4.24) in (3.14), (3.15) and (3.18) to (3.21),

$$\dot{Y}_1(t) = Y_7(t)$$
$$\dot{Y}_2(t) = Y_8(t)$$
$$\dot{Y}_3(t) = Y_9(t)$$
$$\dot{Y}_4(t) = Y_{10}(t)$$
$$\dot{Y}_5(t) = Y_{11}(t)$$
$$\dot{Y}_6(t) = Y_{12}(t)$$

$$\dot{Y}_7(t) = \frac{C_1}{M_1}\left[X_1 - Y_1(t)\right] + \frac{s_1}{M_1}\left[Y_3(t) + L_1 Y_4(t) - Y_1(t)\right] + \frac{D_1}{M_1}\left[Y_9(t) + L_1 Y_{10}(t) - Y_7(t)\right]$$

$$\dot{Y}_8(t) = \frac{C_2}{M_2}\left[X_2 - Y_2(t)\right] + \frac{s_2}{M_2}\left[Y_3(t) + L_2 Y_4(t) - Y_1(t)\right] + \frac{D_2}{M_2}\left[Y_9(t) + L_2 Y_{10}(t) - Y_8(t)\right]$$

$$\begin{aligned}\dot{Y}_9(t) = &-\frac{s_1}{M}\left[Y_3 + L_1 Y_4 - Y_1\right] - \frac{D_1}{M}\left[Y_9 + L_1 Y_{10} - Y_7\right] \\ &- \frac{s_2}{M}\left[Y_3 + L_2 Y_4 - Y_2\right] - \frac{D_2}{M}\left[Y_9 + L_2 Y_{10} - Y_8\right] \\ &- \frac{K_1}{M}\left[Y_3 + l_1 Y_4 - Y_5\right] - \frac{d_1}{M}\left[Y_9 + l_1 Y_{10} - Y_{11}\right] \\ &- \frac{K_2}{M}\left[Y_3 + l_2 Y_4 - Y_6\right] - \frac{d_2}{M}\left[Y_9 + l_2 Y_{10} - Y_{12}\right]\end{aligned}$$

$$\begin{aligned}\dot{Y}_{10}(t) = &-\frac{s_1 L_1}{J}\left[Y_3 + L_1 Y_4 - Y_1\right] - \frac{D_1 L_1}{J}\left[Y_9 + L_1 Y_{10} - Y_7\right] \\ &- \frac{s_2 L_2}{J}\left[Y_3 + L_2 Y_4 - Y_2\right] - \frac{D_2 L_2}{J}\left[Y_9 + L_2 Y_{10} - Y_8\right] \\ &- \frac{K_1 l_1}{J}\left[Y_3 + l_1 Y_4 - Y_5\right] - \frac{d_1 l_1}{J}\left[Y_9 + l_1 Y_{10} - Y_{11}\right] \\ &- \frac{K_2 l_2}{J}\left[Y_3 + l_2 Y_4 - Y_6\right] - \frac{d_2 l_2}{J}\left[Y_9 + l_2 Y_{10} - Y_{12}\right]\end{aligned}$$

$$\dot{Y}_{11}(t) = -\frac{K_1}{m_1}\left[Y_5 - Y_3 - l_1 Y_4\right] - \frac{d_1}{m_1}\left[Y_{11} - Y_9 - l_1 Y_{10}\right]$$

$$\dot{Y}_{12}(t) = -\frac{K_2}{m_2}\left[Y_6 - Y_3 - l_2 Y_4\right] - \frac{d_2}{m_2}\left[Y_{12} - Y_9 - l_2 Y_{10}\right] \tag{4.25}$$

These equations can be numerically integrated. The most widely used fourth order method is the one credited to Gill [29]:

$$Y_{j,i+1} = Y_{j,i} + \frac{h}{6}\left[K_{j,1} + 2\left(1 - \frac{1}{\sqrt{2}}\right)K_{j,2} + 2\left(1 + \frac{1}{\sqrt{2}}\right)K_{j,3} + K_{j,4}\right], \tag{4.26}$$

where

$$K_{j,1} = f_j\,(x, Y_{j,i})$$

$$K_{j,2} = f_j\left(x + \frac{1}{2}h, Y_{1i}, Y_{2i}\, \ldots, Y_{12i}\right)$$

$$K_{j,3} = f_j\left[x + \frac{1}{2}h, Y_{1i} + \left(-\frac{1}{2} + \frac{1}{\sqrt{2}}\right)h K_{1,1} + \left(1 - \frac{1}{\sqrt{2}}\right)h K_{1,2}, \ldots, Y_{12,i} + \left(-\frac{1}{2} + \frac{1}{\sqrt{2}}\right)h K_{12,1} + \left(1 - \frac{1}{\sqrt{2}}\right)h K_{12,2}\right]$$

$$K_{j,4} = f_j\left[x + h, Y_{1,i} - \frac{1}{\sqrt{2}}h K_{12} + \left(1 + \frac{1}{\sqrt{2}}\right)h K_{1,3}, \ldots, -\frac{1}{\sqrt{2}}h K_{12,2} + \left(1 + \frac{1}{\sqrt{2}}\right)h K_{12,3}\right]$$

Using (4.26), equation (4.25) can be integrated. The values of Y_j, $\dot{Y}_j$ and $\ddot{Y}_j$ are stored in each iterations. Subsequently the variance of Y_j, $\dot{Y}_j$, $\ddot{Y}_j$ are determined and compared with those obtained by employing equivalent linearization technique.

4.6 DATA OF A PASSENGER CAR

The information regarding a passenger car, supplied by Hindustan Motors Ltd. [8] are given below :

		Total	Front	Rear
1.	a) Curb weight or unladen	2563 lbs.	1401 lbs.	1162 lbs.
	b) Normal laden (1 Passenger in front, 2 Passengers in rear, + 3 x 20 lbs. luggage)	3043 lbs.	1538 lbs.	1505 lbs.
	c) Fully laden (800 lbs. maximum)	3363 lbs.	1619 lbs.	1742 lbs.
2.	Unsprung mass	-	148 lbs.	276 lbs.
3.	Sprung mass	-	1390 lbs.	1229 lbs.

4. Moment of Inertia of vehicle pitching about its C.G. is 1900 slug ft^2

5. Wheel base 8' 1"

6. Seat location in vehicle :

   Rear seat 16 5/8" away from Rear Axle centre.

   Front seat 48 13/16" away from Rear Axle centre.

The suspension spring and damper characteristics and static load deflection curve for tires are shown in figures (7) to (11).

4.7 RESULTS

The response of two degrees of freedom system is investigated using Equations (4.16) and (4.19). The tire stiffness can not be varied arbitrarily. The weight of unsprung mass at front and rear

does not offer much choice for variation of them. Usually the rear axle is heavier than the front, because it houses the differential etc. The parameters that can be varied are suspension spring stiffness, damping value and portion of sprung mass that each subsystem has to carry. The variable parameters are ω_2 (natural frequency of second mass), ζ_2 (damping coefficient of second mass) and ρ (ratio of mass II and mass I). The ratio of variance of acceleration of mass II to that of acceleration input for varying values of ω_2, ζ_2, ρ is determined using equation (4.16). These values are plotted in figures 13, 14 and 15. Similarly, the ratio of variance of displacement of mass II to displacement input variance are found out by means of equation (4.19). These values are plotted in figures 16, 17 and 18.

The equivalent linear damping of the dashpot with nonlinear characteristics are found out using equation (3.62). An iteration process is used to achieve convergence. The input to the vehicle is known in the form of power spectrum. The road roughness can be varied by changing the expression for power spectrum. This can be achieved by varying the roughness coefficient (see § 2.1.5.). The variation of equivalent linear damping with respect to roughness coefficient is determined and plotted in figure (19). Further, the responses are dependent on the equivalent linear damping. The responses corresponding to different road conditions are plotted in figure(20).

For a known road data, the roughness coefficient can be determined. Complex frequency response for the system is determined

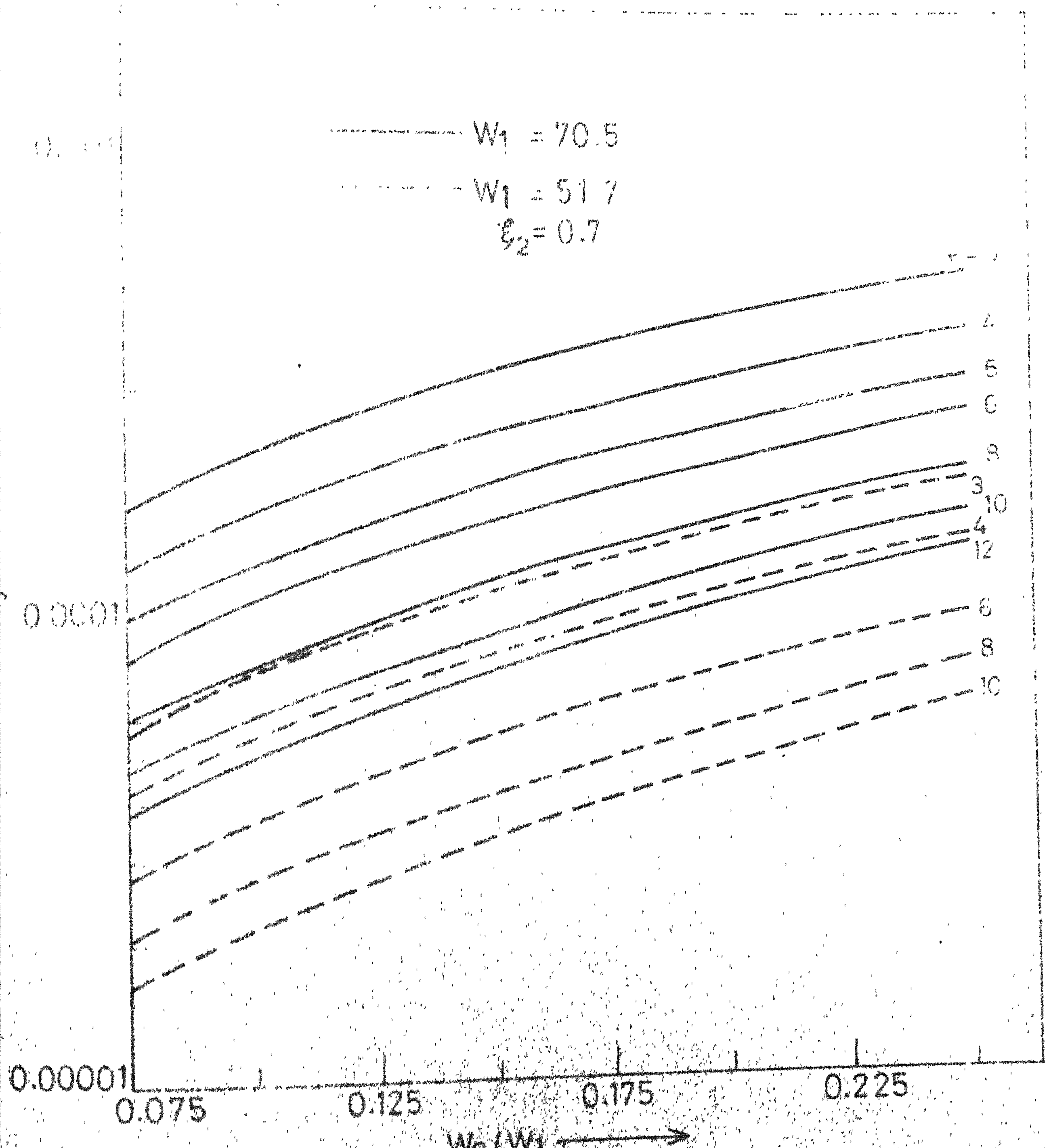

FIG. 13. RATIO OF VARIANCE OF ACCLN. OF MASS II AND OF ACCLN. INPUT vs RATIO OF NATURAL FREQUENCIES OF MASS II AND MASS I.

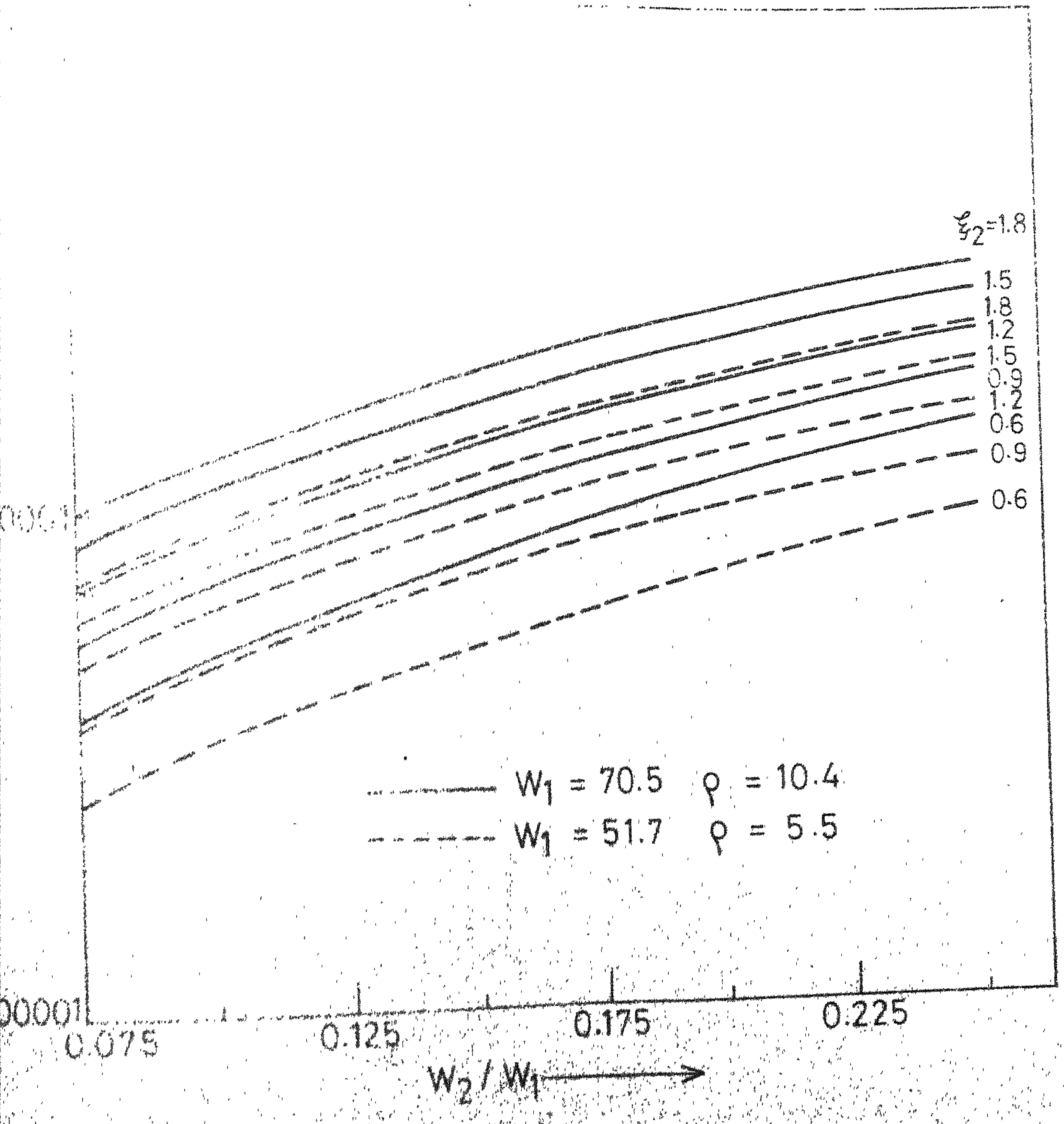

FIG.14. RATIO OF VARIANCE OF ACCLN. OF MASS II AND OF ACCLN. INPUT Vs. RATIO OF NATURAL FREQUENCIES OF MASS II AND MASS I.

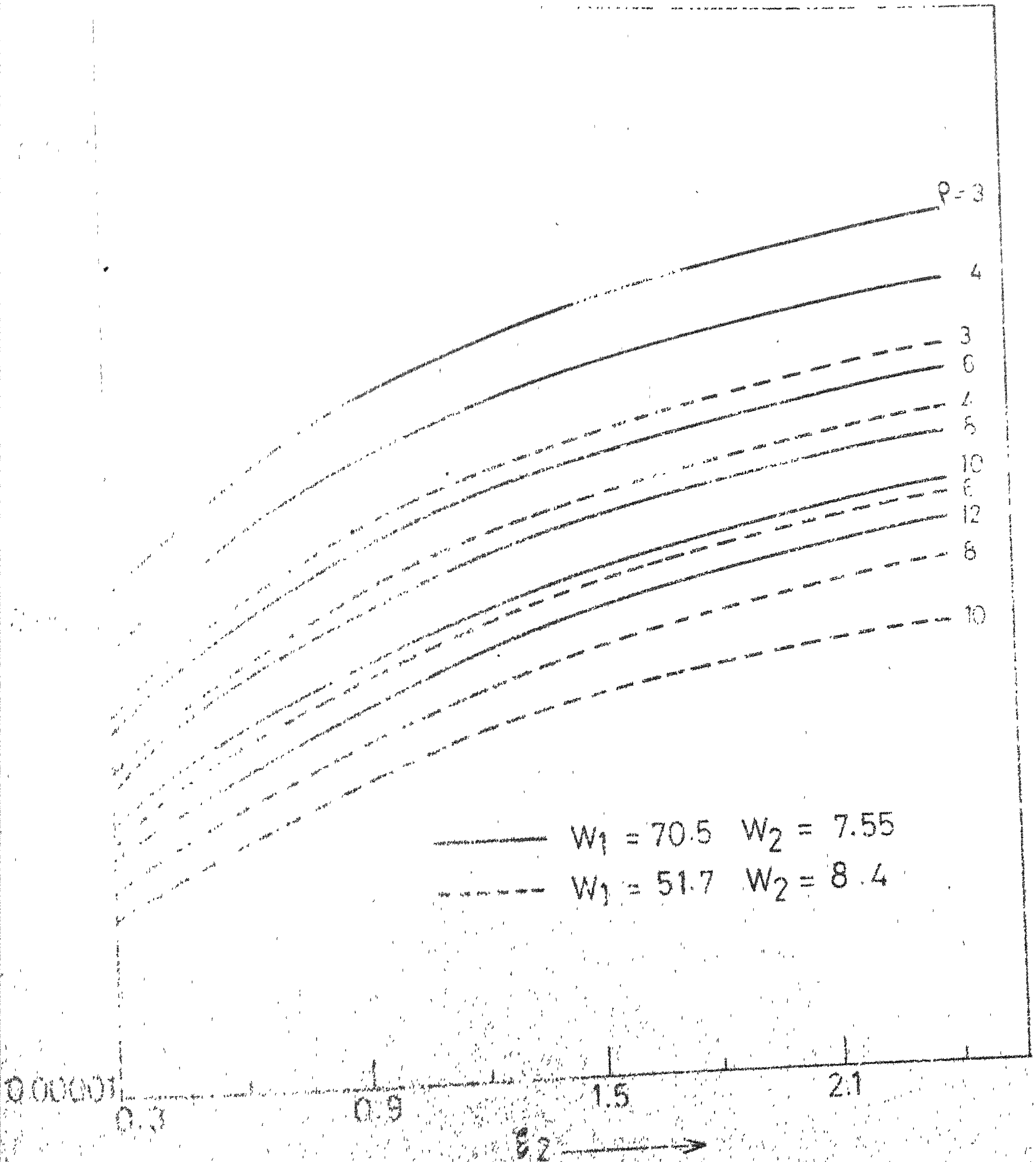

FIG. 15 RATIO OF VARIANCE OF ACCLN. OF MASS II AND OF ACCLN. INPUT Vs DAMPING COEFFICIENT OF MASS II.

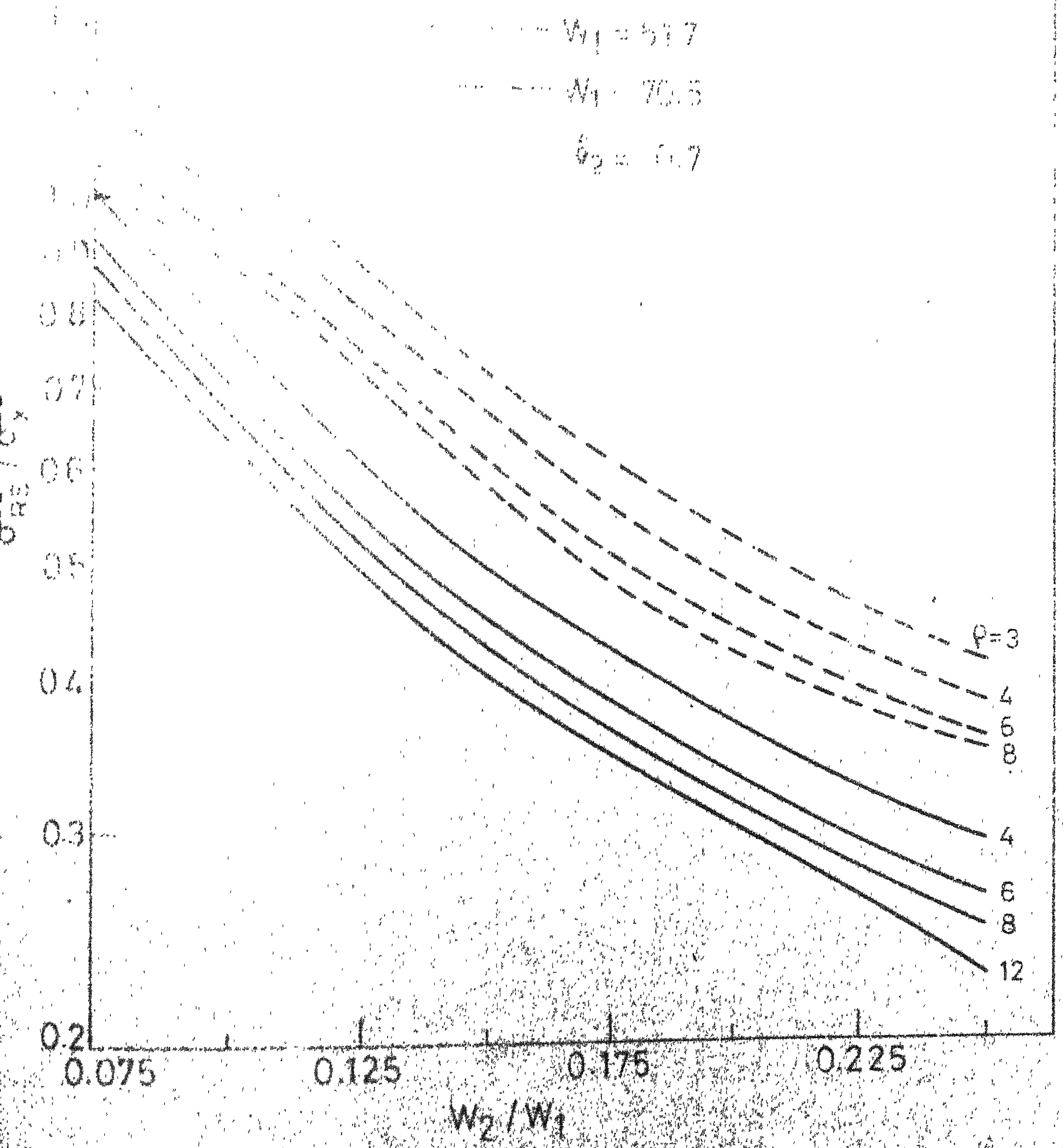

FIG. 16 RATIO OF VARIANCE OF DISPLACEMENT OF MASS II RELATIVE TO MASS I AND OF DISPLACEMENT INPUT Vs. RATIO OF NATURAL FREQUENCY OF MASS II AND MASS I.

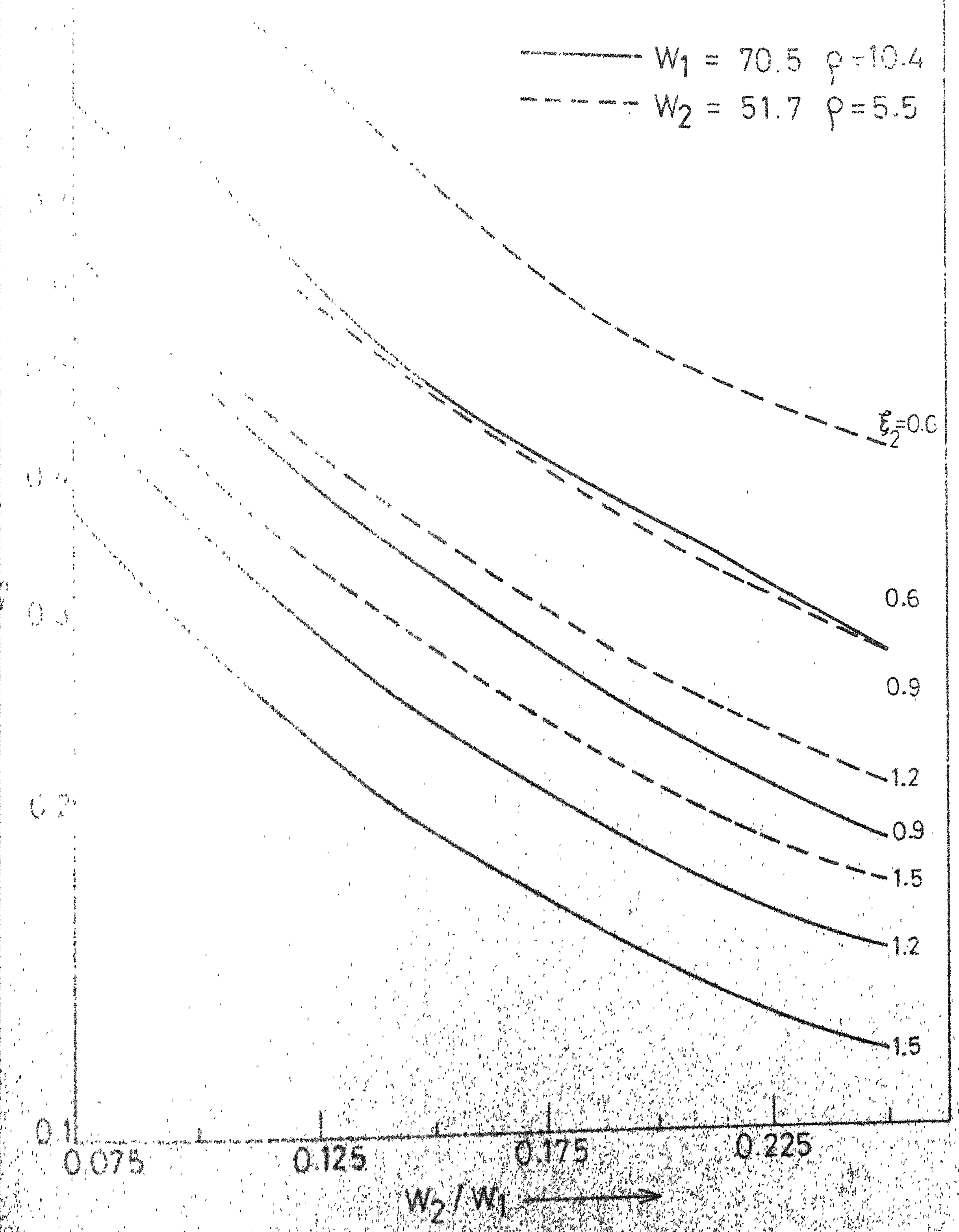

FIG. 17 RATIO OF VARIANCE OF DISPLACEMENT OF MASS II RELATIVE TO MASS I AND OF DISPLACEMENT INPUT Vs. RATIO OF NATURAL FREQUENCY OF MASS II AND MASS I

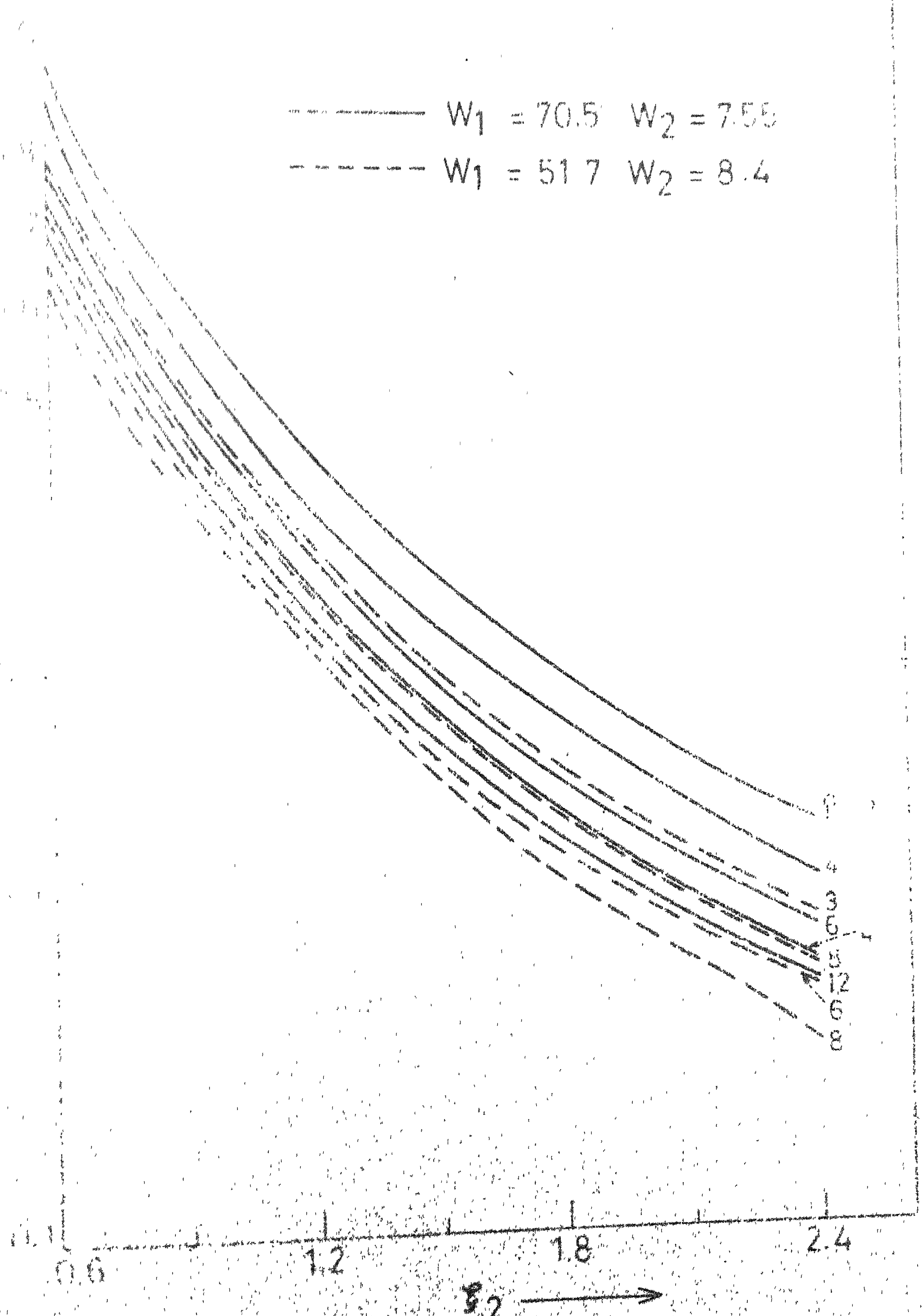

FIG. 18 RATIO OF VARIANCE OF MASS II RELATIVE TO MASS I AND OF DISPLACEMENT INPUT Vs. DAMPING COEFFICIENT OF MASS II.

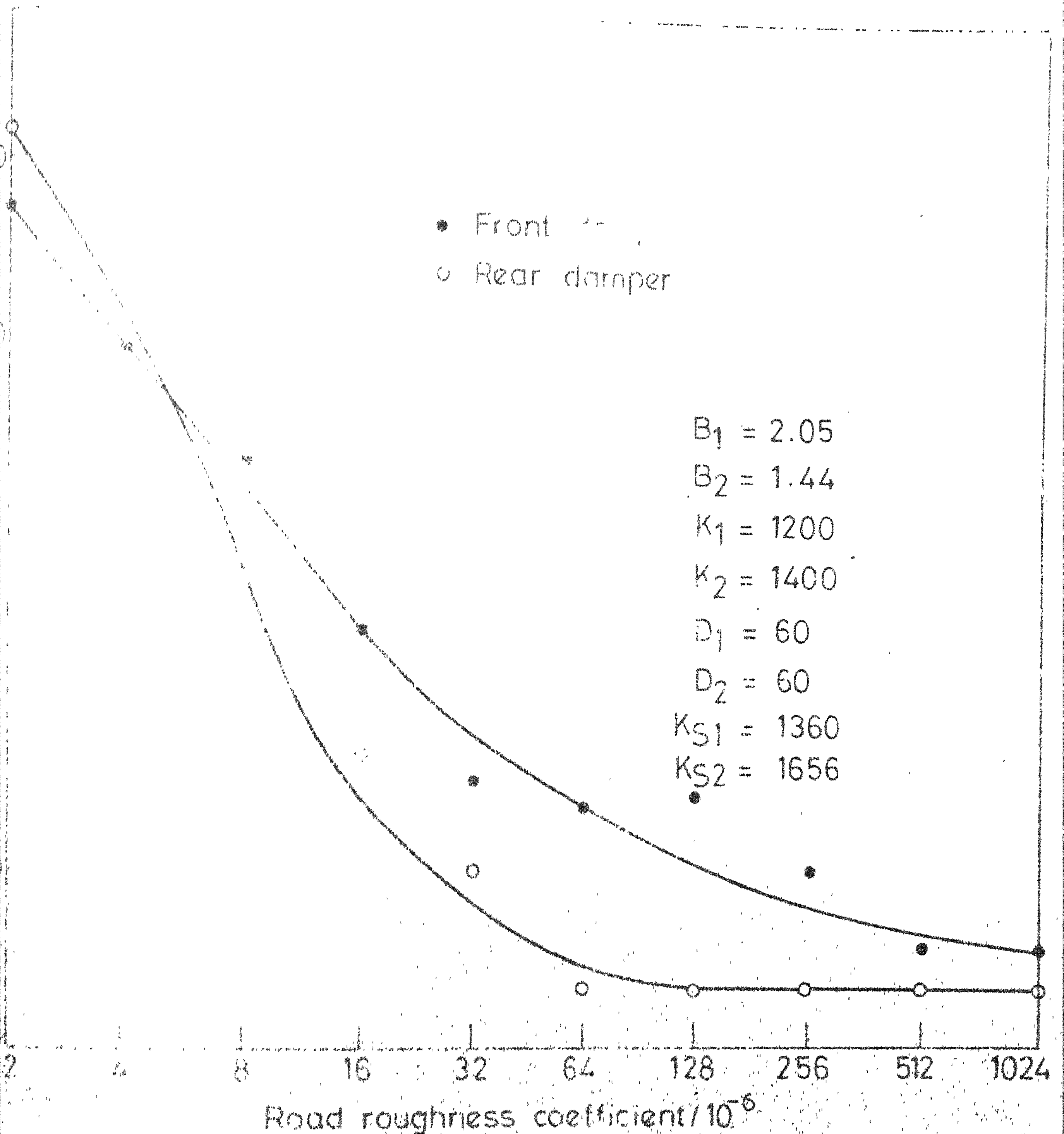

FIG. 10 EQUIVALENT LINEAR DAMPING Vs. ROAD ROUGHNESS COEFFICIENT.

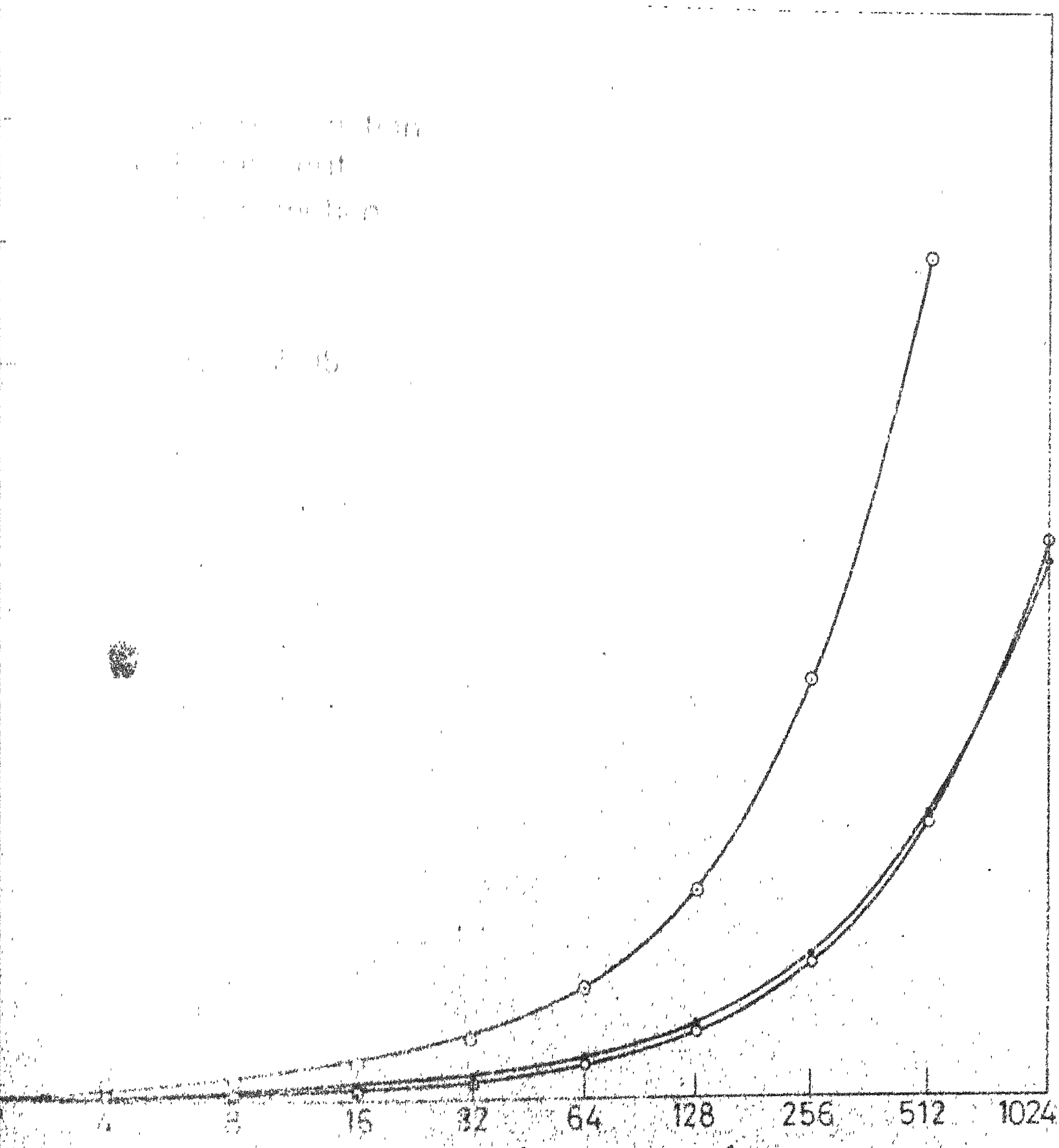

FIG 20 RESPONSES OF THE VEHICLE Vs ROAD ROUGHNESS COEFFICIENT

using equation (3.44). The response of the vehicle for a given road condition can be found out with the help of equation (4.12). There are several parameter which can be varied to change the response quantities. The effect of variation of suspension spring stiffness, suspension damping, seat spring stiffness, seat damping and location of centre of gravity of sprung mass, on response of vehicle are given in Tables 1, 2, 3, 4 and 5 respectively.

Transfer function of human being as a function of frequency is calculated from the I.S.O. proposed recommendation for the effect of vertical vibration on Man (figure 12, 8 hrs curve), using equation (4.22). The power spectrum of the rider response is the product of transfer function of the rider and power spectrum of the seat response. Variance of rider acceleration (corresponding to 4 Hz frequency) can be obtained from the power spectrum of rider response. By varying the suspension parameters, the variance of seat and rider acceleration is varied. The variance of seats and rider acceleration are shown in Table 6 for various seat and damping values.

For a road with roughness coefficient 32×10^{-6}, random data is generated. The method to generate data from power spectrum is discussed in article 2.4. These data are used as input to the system. The system differential equation (4.25) is numerically integrated by Gill's modification of Runge Kutta method. The algorithm for the integration technique is given by equation (4.26). The variance of acceleration of bounce motion is computed and is found to

TABLE 1

EFFECT OF VARIATION OF SUSPENSION SPRING STIFFNESS ON VEHICLE RESPONSES

$C_1 = 1200$ $C_2 = 1400$ $d_1 = 60$ $d_2 = 60$ $D_1 = 100$ $D_2 = 120$

Front suspension spring stiffness lbs./ft.	Rear suspension spring stiffness lbs./ft.	Variance of bounce accln ft.^{2}/sec.^{4}	Variance of pitch accln. Sec.$^{-4}$	Variance of front seat accn. ft.^{2}/sec.^{4}	Variance of rear seat accn. ft^2/sec.^{4}
1100	1100	10.52	.087	3.91	3.98
1100	1300	11.36	.082	4.60	4.76
1100	1500	11.60	.084	4.91	5.29
1300	1100	11.24	.087	4.39	4.25
1300	1300	11.66	.083	5.04	5.11
1500	1100	11.31	.094	4.49	4.38
1500	1300	11.54	.091	4.86	4.86
1500	1500	12.23	.087	5.35	5.76

TABLE 2

EFFECT OF VARIATION OF SUSPENSION DAMPING ON VEHICLE RESPONSES

$s_1 = 1300$ $s_2 = 1500$ $C_1 = 1200$ $C_2 = 1400$ $d_1 = 60$ $d_2 = 60$

Front suspension damping value lb./ft./sec.	Rear suspension damping value lb./ft./sec	Variance of bounce accln ft.^{2}/sec.^{4}	Variance of pitch accln. Sec.$^{-4}$	Variance of front seat accln. ft.^{2}/sec.^{4}	Variance of rear seat accln. ft.^{2}/sec.^{4}
125	100	11.87	.089	5.45	6.01
125	125	12.84	.075	5.23	5.44
125	150	13.61	.074	5.10	5.23
150	125	13.63	.073	5.21	5.35
150	150	14.46	.067	5.13	5.16
150	175	15.26	.068	5.14	5.20
175	150	15.31	.065	5.25	5.13
175	175	16.16	.061	5.30	5.18
175	200	16.97	.063	5.41	5.37
200	175	17.03	.059	5.51	5.18
200	200	17.88	.057	5.64	5.38
200	225	18.68	.058	5.81	5.67

TABLE 3

EFFECT OF VARIATION OF SEAT SPRING STIFFNESS ON THE VEHICLE RESPONSE

$s_1 = 1300$ $s_2 = 1500$ $D_1 = 100$ $D_2 = 120$ $d_1 = 60$ $d_2 = 60$

Front seat spring stiff-ness lbs./ft.	Rear seat spring stiffness lbs.ft.	Variance of bounce accln $ft.^2/sec.^4$	Variance of pitch accln. $Sec.^{-4}$	Variance of front seat accln. $ft.^2/sec.^4$	Variance of rear seat accln. $ft.^2/sec.^4$
1100	1100	11.67	.086	5.42	5.53
1100	1200	11.92	.082	5.47	5.62
1100	1300	11.95	.082	5.46	5.67
1200	1200	11.94	.082	5.54	5.63
1200	1300	11.96	.082	5.53	5.67
1200	1400	11.99	.083	5.53	5.71
1300	1300	11.98	.082	5.60	5.67
1300	1400	12.00	.082	5.60	5.71
1300	1500	12.02	.083	5.59	5.75
1400	1400	12.01	.082	5.67	5.71
1400	1500	12.03	.082	5.66	5.75
1400	1600	12.05	.083	5.65	5.80

TABLE 4

EFFECT OF VARIATION OF DAMPING IN SEAT ON VEHICLE RESPONSE

$s_1 = 1300$ $s_2 = 1500$ $C_1 = 1200$ $C_2 = 1400$ $D_1 = 100$ $D_2 = 120$

Front seat damping lb/ft./sec.	Rear seat damping lb./ft./sec	Variance of bounce accln $ft^2/sec.^4$	Variance of pitch accln. $Sec.^{-4}$	Variance of front seat accln. $ft.^2/sec.^4$	Variance of rear seat accln. $ft.^2/sec.^4$
40	40	12.13	.092	5.86	6.21
60	60	11.99	.082	5.53	5.53
80	80	11.68	.078	5.32	5.42
100	100	11.41	.074	5.24	5.26
120	120	11.16	.071	5.25	5.20
140	140	10.94	.069	5.30	5.20
160	160	10.73	.067	5.39	5.23
180	180	10.54	.064	5.48	5.28
200	200	10.37	.062	5.59	5.36
220	220	10.21	.061	5.71	5.44
240	240	10.06	.060	5.82	5.53

TABLE 5

EFFECT OF LOCATION OF CENTRE OF GRAVITY OF SPRUNG MASS ON THE VEHICLE RESPONSE

$s_1 = 1300$ $s_2 = 1500$ $C_1 = 1200$ $C_2 = 1400$ $D_1 = 100$ $D_2 = 120$
$d_1 = 60$ $d_2 = 60$

Distance of front axle from C.G. of sprung mass ft.	Distnace of rear axle from C.G. of sprung mass ft.	Distance of front seat from C.G. of sprung mass ft.	Distance of rear seat from C.G. of sprung mass ft	Variance of bounce accln. $ft^2/sec.^4$	Variance of pitch accln. $Sec.^{-4}$	Variance of front seat accln. $ft./sec^4$	Variance of rear seat accln. $ft^2/sec.^4$
3	-5	-1	-3.67	11.77	.126	5.27	5.53
3.25	-4.75	-.75	-3.42	12.02	.102	5.49	5.64
3.5	-4.5	-.5	-3.17	12.00	.087	5.53	5.69
3.75	-4.25	-.25	-2.92	11.97	.080	5.52	5.73
4.0	-4.0	.0	-2.67	11.91	.078	5.48	5.73
4.25	-3.75	.25	-2.42	11.84	.083	5.41	5.70
4.5	-3.5	.5	-2.17	11.76	.095	5.32	5.62
4.75	-3.25	.75	-1.92	11.66	.112	5.21	5.50
5.0	-3	1.0	-1.67	11.54	.136	5.10	5.30

TABLE 6

STUDY OF VARIATION OF VARIANCE OF ACCELERATION OF RIDER WITH RESPECT TO VARIANCE OF ACCELERATION OF SEAT

$s_1 = 1300$ $s_2 = 1500$ $C_1 = 1200$ $C_2 = 1400$ $D_1 = 100$ $D_2 = 120$

Front and rear suspension damping lb./ft./sec.	Variance of front seat accln. $ft^2/sec.^4$	Variance of rear seat accln. ft^2/sec^4	Variance of acceleration of rider on front seat. $ft^2/sec.^4$	Variance of acceleration of rider on rear seat. $ft^2/sec.^4$
40	5.87	6.21	3.65	3.86
60	5.53	5.71	3.43	3.53
80	5.32	5.42	3.33	3.38
100	5.24	5.23	3.35	3.33
120	5.25	5.20	3.38	3.35
140	5.30	5.20	3.46	3.39
160	5.39	5.23	3.55	3.46
180	5.49	5.28	3.64	3.54
200	5.59	5.36	3.73	3.63

be 25.89 ft^2/sec^4. Using equivalent linearization technique, the variance of acceleration of bounce is found to be 24.96 ft^2/sec^4. which compares well with that obtained by using numerical integration technique.

4.8 SELECTION OF OPTIMUM PARAMETERS

In the data given in article 4.6, the moment of inertia of sprung mass about its centre of gravity is not known. It is assumed to be 763 slug ft^2. This satisfies the dynamic decoupling condition (refer equation 3.25). After decoupling into two subsystems the following quantities for a passenger car (§ 4.6) are calculated.

TABLE 7

COMPUTED CHARACTERISTICS OF A PASSENGER CAR

	FRONT	REAR
Mass I (m_o)	2.3 slugs	4.28 slugs
Mass II (M_o) (normal laden)	23.9 slugs	23.8 slugs
Natural frequency of Mass I (ω_1)	70.5 rad/sec	51.7 rad/sec
Natural frequency of Mass II (ω_2)	7.55 rad/sec	8.4 rad/sec.
Ratio of Mass II to Mass I (ρ)	10.4	5.55
Ratio of natural frequencies of Mass II to that of Mass I (ω_2/ω_1)	.107	.162

If the vehicle moves on a road at 60 m.p.h. with roughness coefficient 32 x 10^{-6} (equation 2.17), the damping values for

front and rear wheels are found to be 250 and 201 lb/ft/sec respectively (using figure 19). The corresponding damping coefficients can be calculated, using equation 3.33, and are found to be .7 and .51 respectively.

From the figure 14, the ratio of variance of acceleration of mass II to variance of acceleration of disturbance input ($\sigma_{\ddot{y}}^2/\sigma_{\ddot{x}}^2$) are found to be .56 x 10^{-4} and .52 x 10^{-4}. The frequency range of interest for human vibration is 1 to 80 cps. Integrating the power spectrum of displacement road input (using equation 2.27) in this frequency range the variance of input acceleration ($\sigma_{\ddot{x}}^2$) is found to be 2.885 x 10^5 $ft^2/sec.^4$. Therefore, the standard deviation of acceleration of mass II ($\sigma_{\ddot{y}}$) of front and rear portions are 4 and 3.86 respectively.

Similarly the standard deviation of relative displacement of mass II with respect to Mass I (σ_{RS}) are found(refer figure 17) to be .208" and .224". If the clearance be taken as 5 σ_{RS} value which turns out to be 1.04" and 1.12" respectively, the probability that the displacement of mass II relative to mass I will be within these values is 0.9999997. But in actual vehicle the clearance provided for the front and rear suspensions are 4" and 3.75" in normal laden condition and 3.5" and 3" in fully laden condition (figure 7 and 8) which are far above even the safe 5 σ_{RS} values. Thus the clearance seems to be overdesigned.

It can be seen that for the actual bump stop distance provided, the vehicle can move on a more rough road at the speed of 60 m.p.h. e.g. an average principal road with roughness coefficient of 128×10^{-6} [4].

It is not desirable that the spring should be compressed to its solid length. The 5 σ_{RS} value is taken to be 2.5". The variance of displacement of mass II relative to mass I is given by,

$$\sigma_{RS}^2 = \left(\frac{2.5}{5 \times 12}\right)^2 = .00174 \text{ ft}^2.$$

The variance of displacement for the road roughness can be determined by integrating power spectrum for displacement road input (using equations 2.6 and 2.17) in the frequency range of 1 to 80 Hz. This is found to be .00187 ft^2. Therefore,

$$\frac{\sigma_{RS}^2}{\sigma_X^2} = \frac{.00174}{.00187} = .85$$

For the values, $\zeta_2 = .6$, $f = 10.4$ and 5.5 for the front and rear portions, the corresponding ω_2 / ω_1 are found from figure 17. These values are .0875 and .13, corresponding to ω_1 values of 70.5 and 51.7 (table 7), the values of ω_2 are 6.17 and 6.75 rad/sec respectively. Using equation (3.33) the front and rear suspension stiffness are calculated to be 920 and 1090 lbs/ft.

Many of the data required such as seat spring and dashpot characteristics are not available. If these data are known, the optimum parameters for maximum comfort of rider can be determined. The effect of variation of different parameters on the system performance will be discussed in the next chapter.

CHAPTER V

CONCLUSIONS

5.1 GENERAL OBSERVATION AND COMMENTS

From results obtained in the previous chapters, following observations and comments can be made :

(i) The acceleration of Mass II (Fig. 5) increases as ω_2 / ω_1 increases (Fig. 13, 14). Keeping ω_1 constant, acceleration of mass II increases as ω_2 increases. From equation (3.33), it can be observed that acceleration increases as stiffness increases. Since mass II does not offer much flexibility for a change, the decrease in acceleration can be achieved by decreasing the stiffness. But the decrease in stiffness causes increase in the displacement of mass II relative to mass I (Fig. 16, 17 and 18). Since the bump stop distance is fixed, the stiffness of suspension can not be decreased arbitrarily. If the spring is compressed to its solid length, the spring acts as a rigid link between axles and chassis. Thus the rider is subject to a very high value of acceleration causing discomfort.

(ii) Acceleration of mass II increases as damping coefficient ξ_2 increases (Fig. 15). Thus for better comfort it is recommended to have as low damping as permitted. But damping can not be lowered below the optimum road holding values [16].

(iii) Figure 15 shows that as ρ (ratio of mass II to mass I) decreases, the acceleration of mass II increases. There is little scope for varying the unsprung mass I but mass II can be varied to a certain extent by placing the centre of gravity of sprung mass so that equal acceleration is obtained at both front and rear portions of the vehicle. However, the position of C.G. between the two axles can not be varied to a great extent and thus it appears that the parameter ρ does not offer much scope for variation.

(iv) The suspension damping characteristics are nonlinear in nature. The equivalent linear damping value is determined applying equivalent linearization technique (equation 3.62). From figure (19) it can be observed that the damping value decreases as the road roughness coefficient is increased. Equation 3.63 confirms this result. Physically it can be explained as follows. The relative velocity between sprung and unsprung mass increases as the road roughness increases. The damping in shock absorber consists of linear viscous damping and Coulomb friction. Although the linear viscous damping force increases proportionately with relative velocity, Coulomb friction does not increase correspondingly. Thus the overall damping force does not increase proportionately to road roughness. At high value of road roughness, the friction force in dashpot is very small compared to the linear viscous damping force. Therefore, the damping value remains almost constant.

As the road roughness is doubled, the acceleration response of sprung mass and seats does not increase two fold (Fig. 20). This is due to decrease in damping force. However at higher values of road roughness it increases proportionately since the damping value remain almost constant.

(v) As expected, if the spring stiffness of front and rear suspension is increased, the bounce mode increases (Table 1), but the pitch motion is not much affected.

(vi) As damping in suspension increases, the bounce mode increases (Table 2) because as damping increases more of disturbance is transmitted.

(vii) As seat spring stiffness increases, bounce acceleration and acceleration of seats increases (Table 3). It is desirable to employ a spring with a low stiffness. The criterion could be fixed by a given static deflection when the rider sits on it.

(viii) As seat damping is increased from a very low value, acceleration of sprung mass and seat decreases (Table 4). If the seat damping is further increased beyond a certain value the seat acceleration increases. This is because for low values of damping, an increase in damping value reduces the oscillations of seats where as for high values of damping, an increase in damping helps in transmitting more of vibration from sprung mass to seat.

(ix) If the centre of gravity of sprung mass is shifted from centre of axles, the bounce and seat acceleration may decrease slightly as can be seen from table 5, but pitch acceleration increases. Because pitch motion causes more discomfort than bounce, the centre of gravity should be as near as possible to the centre of two axles. An increase in wheel base will cause decrease in pitch oscillation.

(x) The equivalent acceleration of rider at 4 Hz is calculated according to equation (4.21). From table 6, it can be seen that the variance of acceleration of rider varies as variance of acceleration of seat. Therefore the former need not be calculated except where it is desired to design the seat spring and to know the comfort conditions.

5.2 AVENUES FOR FURTHER RESEARCH

For off road conditions, the assumption regarding stationarity of ground profile may not be justified. In such cases, the concept of continuously changing power spectrum or "evolutionary power spectrum" can be used. Expressions for the response quantities for the vehicle, can be developed in terms of the evolutionary spectrum of the terrain.

There are various sources of inputs to the passenger and driver. For instance vibrations are transmitted to the driver through steering besides seat and foot rest. If the vibration transmitted through the steering column can be minimized, the handling characterstics of the car can be considerably improved even for rough roads.

To improve the ride comfort, study can be made of the vehicle with seat and foot rest seperated from the chassis by spring and dashpots.

Planes that land on aircraft carriers get a short runway distance. They are subject to very high decelerations. Therefore, the responses momentarily spurts up to very high value due to deceleration of the vehicle. This case needs further study.

In this investigation, the effect of engine vibration has been neglected. Engine transmits periodic disturbing forces to the chasis. At certain speeds, such as idling, the frequency of disturbing forces due to engine may coincide with resonant frequency of vehicle. In this case the disturbing forces due to engine, however small it may be, affect the comfort of passengers. The disturbing forces due to wind gust and other aerodynamic forces may be investigated.

Effect of vibration on man has been studied for determinastic inputs, but random vibration techniques has not been properly attempted. An extensive study of human comfort should be done and mental and physical fatigue criteria need be established more realistically.

Response of large vehicles carrying vibration sensitive cargo or human beings and having more than 4 wheels should be studied. This extends to buses, large trucks and ambulances etc.

Off road vehicles, such as tanks, military vehicles etc., need be properly analysed for rough terrain profiles. The criteria in these cases might not be comfort, but of proper tolerances, ability to align guns to the target or for radar detection etc.

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